Chapter **11** (Benson)

E06 $a_t = r\alpha$, $a_c = r\omega^2$, $a^2 = a_t^2 + a_c^2 = r^2(\alpha^2 + \omega^4)$.

E10 (a) $\omega = 2\pi/60 = 0.105$ (rad/s); (b) $v = r\omega = (0.08)(0.105) = 8.38\times10^{-3}$ (m/s) .

E23 (a) $I = M(2d)^2 + 2M(\sqrt{2}d)^2 = 8Md^2$; (b) $I = 2Ml^2 + M(\sqrt{2}l)^2 = 4Ml^2$.

E29 $I_{rod} = m(3R)^2/12 = 0.75\,mR^2$, $I_s = 2\,mR^2/5 + m(2.5R)^2 = 6.65\,mR^2$, $I = I_{rod} + 2I_s = 14.05\,mR^2$.

E31 $I = (2\pi Rh\sigma)R^2 + (2)(\pi\sigma R^2/2)R^2 = \pi\sigma R^3(2h + R)$.

E32 Distance of elements to axis reduced from x to $x\sin\alpha$, so $ML^2/3$ becomes $ML^2\sin^2\alpha/3$.

E34 (a) $I_x = m\,y^2$, $I_y = m\,x^2$; (b) $I_z = m\,r^2 = I_x + I_y$.

E39 $\Delta E = 0$, $mv^2/2 + I\,(v/R)^2/2 + kx^2/2 = mgx$. (a) $v = 0$, $x = 2mg/k = (2)(4)(9.8)/80 = 0.98$ (m); (b) $(4)(v^2/2) + (1)(v^2/2) = (4)(9.8)(0.20) - (80)(0.20)^2/2$, $v^2 = 2.496$, $v = 1.58$ (m/s) .

E43 $E_i = mgR$, $E_f = I\omega^2/2$, where $I = 3MR^2/2$. Thus $\omega = \sqrt{4g/3R}$, $v = \omega\,(2R) = \sqrt{16gR/3}$.

E44 $m_r = 1.2$ kg, $m_d = 0.40$ kg, $l = 0.60$ m, $R = 0.05$ m & $\theta = 30^0$. (a) $I = M_r l^2/3 + M_d (l+R)^2 + M_d R^2/2 = 0.314$ (kg.m^2); (b) $E_i = E_f$, $[M_r gl/2 + M_d g(l+R)](1-\cos\theta) = I\omega^2/2$, $(0.314)\omega^2/2 = 0.814$, $\omega = 2.28$ (rad/s) .

E56 (a) $Mgh = (3MR^2/2)\omega^2/2 \rightarrow v = \sqrt{4gh/3}$; (b) $v^2 = 2gh$, thus $a = 2g/3$; (c) Take τ about the point of contact with the string. $\tau = I\alpha$: $mgR = (3MR^2/2)\ (a/R)$, thus $a = 2g/3$; (d) $mg - T = ma$, thus $T = mg/3$; (e) $a = 0$ leads to $T = mg$. Take τ about the center: $\tau = I\alpha$: $TR = (MR^2/2)\,\alpha$, leads to $\alpha = 2g/R$. (Teacher: Jyh-Shinn Yang, 89.12.22)

E58 (a) $F - f_s = Ma_c$ & $FR = I_p\alpha = (3MR^2/2)\alpha$, $\alpha = 2F/(3MR)$, $a_c = R\alpha = 2F/(3M)$; (b) $f_s = F - Ma_c = F/3$; (c) $f_s \le \mu_s mg$ leads to $F \le 3\mu_s mg$.

E59 (a) $mg\sin\theta - f_s = ma_c$; $(mg\sin\theta)R = I_p\alpha$, where $I_p = 3mR^2/2$. $\alpha = 2g\sin\theta/3R$, $a_c = R\alpha = 2g\sin\theta/3$; (b) $f_s = mg\sin\theta - ma_c = mg\sin\theta/3$. $f_s \le \mu_s N = \mu_s(mg\cos\theta)$, $\mu_s \ge \tan\theta/3$.

E66 (a) $W = \Delta K = I\omega^2/2 = MR^2\omega^2/4 = (7600)(2\pi/30)^2 = 385$ (J);
(b) $W = Fd$, $d = W/F = 385/1.2 = 321$ (N) .

E71 $m_1 = 1.2$ kg, $m_2 = 1.8$ kg, $d = 0.4$ m & $M = 1.6$ kg.
$(m_1 + m_2)v^2/2 + (MR^2/2)(v/R)^2/2 = (m_2 - m_1)gd$, thus $v = 1.11$ (m/s) .

P02 Cons. of E: $mgH = mv^2/2 + (2Mr^2/5)(v/r)^2/2 + mg(2R)$, $v^2 = 10g(H - 2R)/7$. At top point, $mg + N = mv^2/R$, $v^2 \ge gR$, so $H \ge 27R/10$.

P03 (a) $E_i = E_f$, $mgL/2 = .5mv_c^2 + .5I_{cm}\omega^2$, where $I_{cm} = mL^2/12$. On landing $v_l = 0$, $v_c = \omega(L/2)$, thus $\omega^2 = 3g/L$; (b) $v_l = 0$, $v_r = \omega L/2 = \sqrt{3gl/4}$.

P04 $t = \sqrt{2h/g}$. Relative to earth's surface $V_{rel} = \omega\,(h + R) - \omega R = \omega h$, thus $d = V_{rel}\,t =$

$\omega h\sqrt{2h/g}$.

P06 (a) $r = R\cos\theta$, $dm = 2\pi\sigma(Rd\theta) = 2\pi\sigma R^2\cos\theta d\theta$. $I = \int r^2\,dm = 2\pi\sigma R^4\int_{-\pi/2}^{\pi/2}\cos^3\theta\,d\theta = 8\pi\sigma R^4/3$. Use $M = 4\pi\sigma R^2$ to obtain $I = 2MR^2/3$; (b) $I = \int dI = \int(8\pi\rho r^4/3)dr = 8\pi\rho R^5/15$. Use $M = 4\pi\rho R^3/3$ to obtain $I = 2MR^2/5$.

P09 Let the apex of the cone be at the origin. $M = \int dm = \int 2\pi\sigma r\,dz\csc\alpha$, where $r = z\tan\alpha$. $M = 2\pi\sigma\tan\alpha\csc\alpha h^2/2$. $I = \int r^2\,dm = \int 2\pi\sigma\tan^3\alpha\csc\alpha z^3 dz = 2\pi\sigma\tan^3\alpha\csc\alpha h^4/4$. Thus $I = M(h\tan\alpha)^2/2$.

P12 Same as that of E.11-59 .

P13 $I_x = Ma^2/12$, $I_y = Mb^2/12$, thus $I_z = M(a^2+b^2)/12$. (Teacher: Jyh-Shinn Yang, 89.12.22)