Chapter **2** (Benson)

E19 (a) $A = 2\,\text{km}$, $\theta_A = 45^\text{o}$; $B = 1.5\,\text{km}$, $\theta_B = 15^\text{o}$; $C = 1.5\,\text{km}$, $\theta_C = -105^\text{o}$. So $\vec{A} = (1.41\,\hat{\imath} + 1.41\,\hat{\jmath})\text{km}$, $\vec{B} = (1.45\,\hat{\imath} + 0.39\,\hat{\jmath})\,\text{km}$, and $\vec{C} = (-0.39\,\hat{\imath} - 1.45\,\hat{\jmath})\,\text{km}$. We find $\vec{D} = -(\vec{A}+\vec{B}+\vec{C}) = (-2.47\,\hat{\imath} - 0.35\,\hat{\jmath})\,\text{km}$; (b) 2.49 km at 8.07^o S of W.

E22 Let $\vec{R} = \vec{A} + \vec{B} + \vec{C}$, where $\vec{A} = 40\,\hat{\jmath}\,\text{km}$, $\vec{B} = 30\,\hat{\imath}\,\text{km}$, and $\vec{R} = -17.3\,\hat{\imath} - 10\,\hat{\jmath}$. So $\vec{C} = (12.7\,\hat{\imath} - 50\,\hat{\jmath})\,\text{km}$, or $C = 51.6\,\text{km}$ at 75.7^o S of E.

E23 $\vec{C} = \vec{A} + (\vec{B} - \vec{A})/2 = (\vec{A} + \vec{B})/2$.

E39 $\cos\theta = \vec{A}\cdot\vec{B}/AB = (-4)/(5\times 13)^{1/2}$, so $\theta = 120^\text{o}$.

E40 $\vec{A} = -2\,\hat{\imath} + \hat{\jmath} - 3\,\hat{k}$ & $\vec{B} = 5\,\hat{\imath} + 2\,\hat{\jmath} - \hat{k}$ (a) $\vec{A}\cdot\vec{B} = -10 + 2 + 3 = -5$; (b) $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = A^2 - B^2 = -16$.

E44 (a) $\vec{A} + \vec{B}$, and $\vec{A} - \vec{B}$ or $\vec{B} - \vec{A}$; (b) $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = 0$ if $A = B$.

E45 $\vec{A}\cdot\hat{\imath} = A_x = A\cos\alpha$, thus $\cos\alpha = \vec{A}\cdot\hat{\imath}/A$, etc. $\vec{A} = 3\,\hat{\imath} + 2\,\hat{\jmath} + \hat{k}$, $\alpha = \cos^{-1}(3/\sqrt{14}) = 36.7^\text{o}$; $\beta = \cos^{-1}(2/\sqrt{14}) = 57.7^\text{o}$; $\gamma = \cos^{-1}(1/\sqrt{14}) = 74.5^\text{o}$.

E46 $\vec{A} = \hat{\imath} - 4\,\hat{\jmath}$, $\vec{B} = 3\,\hat{\imath}$ & $\vec{C} = -2\,\hat{\jmath}$. (a) $\vec{C}\cdot(\vec{A}+\vec{B}) = 8$; (b) Not allowed; (c) $C + (\vec{A}\cdot\vec{B}) = 5$; (d) $C(\vec{A}\cdot\vec{B}) = 6$; (d) $\vec{C}(\vec{A}\cdot\vec{B}) = -6\,\hat{\jmath}$.

E48 $\vec{A} = \hat{\imath} + 2\,\hat{\jmath} - 4\,\hat{k}$ & $\vec{B} = 3\,\hat{\imath} - \hat{\jmath} + 5\,\hat{k}$. $\vec{A}\times\vec{B} = 6\,\hat{\imath} - 17\,\hat{\jmath} - 7\,\hat{k}$.

E49 (a) $\vec{A}\cdot(\vec{A}\times\vec{B}) = 0$; (b) $\vec{A}$ is perpendicular to $\vec{A}\times\vec{B}$.

E51 Area = Base × Height = $(B)(A\sin\theta) = |\vec{A}\times\vec{B}|$.

E52 $\vec{A} = 2\,\hat{\imath} - 5\,\hat{\jmath}$, $\vec{B} = 4\,\hat{\jmath}$ & $\vec{C} = 3\,\hat{\imath}$. (a) $C(\vec{A}\times\vec{B}) = 24\,\hat{k}$; (b) $\vec{C}\cdot(\vec{A}\times\vec{B}) = 0$; (c) $\vec{C}\times(\vec{A}\cdot\vec{B})$ not allowed; (d) $\vec{C}\times(\vec{A}\times\vec{B}) = -24\,\hat{\jmath}$; (e) $\vec{C} + \vec{A}\times\vec{B} = 3\,\hat{\imath} + 8\,\hat{k}$.

E54 Find $\hat{n} = (\vec{A}\times\vec{B})/(AB\sin\theta) = (14\,\hat{\imath} + 19\,\hat{\jmath} - \hat{k})/(23.6)$. Thus $\vec{C} = 5\,\hat{n} = 2.96\,\hat{\imath} + 4.02\,\hat{\jmath} - 0.21\,\hat{k}$.

E65 $A = B$, $\theta_A = 30^\text{o}$, $\vec{B} \perp \vec{A}$, & $|\vec{A} + \vec{B}| = 2.12\,\text{m}$. (a) $|\vec{A} + \vec{B}|^2 = A^2 + B^2 = 2,12^2$, so $A = B = 1.50\,\text{m}$; (b) $\theta_B = 120^\text{o}$, $\vec{C} = \vec{A} + \vec{B} = (0.549\,\hat{\imath} + 2.05\,\hat{\jmath})\,\text{m}$; (c) $\theta_B = -60^\text{o}$, $\vec{C} = (2.05\,\hat{\imath} - 0.549\,\hat{\jmath})\,\text{m}$. (Teacher: Jyh-Shinn Yang, 89.10.06)

P04 (a) $\vec{r} = x\,\hat{\imath} + y\,\hat{\jmath} = r\cos\phi\,\hat{\imath} + r\sin\phi\,\hat{\jmath} = x'\,\hat{\imath}' + y'\,\hat{\jmath}'$. $\hat{\imath}' = \cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}$ & $\hat{\jmath}' = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$, or $\hat{\imath} = \cos\theta\,\hat{\imath}' - \sin\theta\,\hat{\jmath}'$ & $\hat{\jmath} = \sin\theta\,\hat{\imath}' + \cos\theta\,\hat{\jmath}'$. Thus $x' = r\cos\phi\cos\theta + r\sin\phi\sin\theta = x\cos\theta + y\sin\theta$, $y' = -r\cos\phi\sin\theta + r\sin\phi\cos\theta = -x\sin\theta + y\cos\theta$.

P05 (a) $\vec{D} = \hat{\imath} + \hat{\jmath} + \hat{k}$. $\cos\alpha = \vec{D}\cdot\hat{k}/D = 1/\sqrt{3}$, $\alpha = 54.7^\text{o}$; (b) Take $\vec{A} = \hat{\imath} + \hat{\jmath}$ & $\vec{B} = \hat{\imath} + \hat{k}$, thus $\vec{A}\cdot\vec{B} = 1$, $\beta = 60^\text{o}$; (c) $\cos\gamma = \vec{A}\cdot\vec{D}/AD = 2/\sqrt{6}$, so $\gamma = 35.3^\text{o}$.

P07 We know that area of base = $|\vec{B}\times\vec{C}|$, where the vector $\vec{B}\times\vec{C}$ is along $\hat{n}$, the normal to the plane of $\vec{A}$ and $\vec{B}$. The height of the parallelopiped is $H = A\cos\beta$, where β is the angle between $\vec{A}$ and $\hat{n}$, thus $H = \vec{A}\cdot\hat{n}$. The volume is V = Height × Base =

$(\vec{A}\cdot\hat{n})|\vec{B}\times\vec{C}| = \vec{A}\cdot(\vec{B}\times\vec{C})$.

P09 $\hat{r} = \cos\theta\,\hat{\mathrm{i}} + \sin\theta\,\hat{\mathrm{j}}$. Since $\hat{\theta} \perp \hat{r}$, So $\hat{\theta} = -\sin\theta\,\hat{\mathrm{i}} + \cos\theta\,\hat{\mathrm{j}}$.

P10 $\cos\alpha = \vec{r}\cdot\hat{\mathrm{i}}/A = x/r$, $\cos\beta = \vec{r}\cdot\hat{\mathrm{j}}/A = y/r$ & $\cos\gamma = \vec{r}\cdot\hat{\mathrm{k}}/A = z/r$. We have $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = (x^2 + y^2 + z^2)/r^2 = 1$. (Teacher: Jyh-Shinn Yang, 89.10.06)