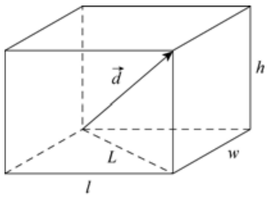
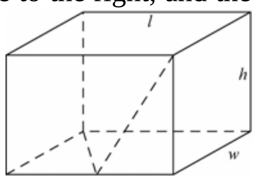
Chapter **3** *Vector Quantities*

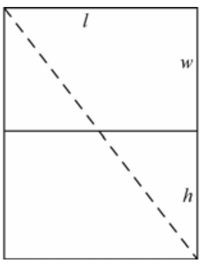
07. The length unit meter is understood throughout the calculation. (**a**) We compute the distance from one corner to the diametrically opposite corner:

$$d = (3.00^2 + 3.70^2 + 4.30^2)^{1/2} = 6.42\text{ m}.$$

(**b**) The displacement vector is along the straight line from the beginning to the end point of the trip. Since a straight line is the shortest distance between two points, the length of the path cannot be less than the magnitude of the displacement. (**c**) It can be greater, however. The fly might, for example, crawl along the edges of the room. Its displacement would be the same but the path length would be $\ell + w + h$ = 11.0 m. (**d**) The path length is the same as the magnitude of the displacement if the fly flies along the displacement vector. (**e**) We take the x axis to be out of the page, the y axis to be to the right, and the z axis to be upward. Then the x component of the displacement is w = 3.70, the y component of the displacement is 4.30, and the z component is 3.00. Thus $\vec{d} = 3.70\,\hat{\mathrm{i}} + 4.30\,\hat{\mathrm{j}} + 3.00\,\hat{\mathrm{k}}$. An equally correct answer is obtained by interchanging the length, width, and height. (**f**) Suppose the path of the fly is as shown by the dotted lines on the upper diagram. Pretend there is a hinge where the front wall of the room joins the floor and lay the wall down as shown on the lower diagram. The shortest walking distance between the lower left back of the room and the upper right front corner is the dotted straight line shown on the diagram. Its length is







$$L_{min} = \sqrt{(w+h)^2 + \ell^2} = 7.96\text{ (m)}.$$

$$\langle\ = \sqrt{(3.70+3.00)^2 + 4.30^2}\ \rangle$$

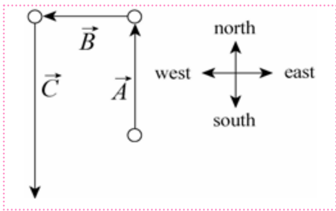
13. All distances in this solution are understood to be in meters. (**a**) $\vec{a} + \vec{b} = [4.0 + (-1.0)]\,\hat{\mathrm{i}} + [(-3.0) + 1.0]\,\hat{\mathrm{j}} + (1.0{+}4.0)\,\hat{\mathrm{k}} = 3.0\,\hat{\mathrm{i}} - 2.0\,\hat{\mathrm{j}} + 5.0\,\hat{\mathrm{k}}$. (**b**) $\vec{a} - \vec{b} = [4.0 - (-1.0)]\,\hat{\mathrm{i}} + [(-3.0) - 1.0]\,\hat{\mathrm{j}} + (1.0 - 4.0)\,\hat{\mathrm{k}} = 5.0\,\hat{\mathrm{i}} - 4.0\,\hat{\mathrm{j}} - 3.0\,\hat{\mathrm{k}}$. (**c**) The requirement $\vec{a} - \vec{b} + \vec{c} = 0$ leads to $\vec{c} = \vec{b} - \vec{a}$, which is the opposite of what we found in part (b). Thus, $\vec{c} = -5.0\,\hat{\mathrm{i}} + 4.0\,\hat{\mathrm{j}} + 3.0\,\hat{\mathrm{k}}$.

10. We label the displacement vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ (and denote the result of their vector sum as $\vec{r}$). We choose *east* as the $\hat{\mathrm{i}}$ direction (+x axis) and *north* as the $\hat{\mathrm{j}}$ direction (+y axis). All distances are understood to be in kilometers. (**a**) The vector diagram representing the motion is shown below. (**b**) The final point is represented by

$$\vec{r} = \vec{A} + \vec{B} + \vec{C} = -2.4\,\hat{\mathrm{i}} - 2.1\,\hat{\mathrm{j}}.$$

whose magnitude is

$$|\,\vec{r}\,| = \sqrt{(-2.4)^2 + (-2.1)^2} \approx 3.2\text{ (km)}.$$



$\vec{A} = 3.1\,\hat{\mathrm{j}}$

$\vec{B} = -2.4\,\hat{\mathrm{i}}$

$\vec{C} = -5.2\,\hat{\mathrm{j}}$

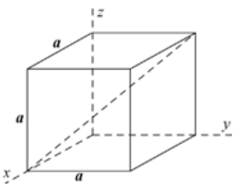
(**c**) There are two possibilities for the angle:

$$\tan^{-1}\left(\frac{-2.1}{-2.4}\right) = 41°,\text{ or } 221°.$$

We choose the latter possibility since $\vec{r}$ is in the third quadrant. It should be noted that many graphical calculators have polar ↔ rectangular "shortcuts" that automatically produce the correct answer for angle (measured counterclockwise from the +x axis). We may phrase the angle, then, as 221° counterclockwise from East (a phrasing that sounds peculiar, at best) or as 41° south from west or 49° west from south. The resultant $\vec{r}$ is not shown in our sketch; it would be an arrow directed from the "tail" of $\vec{A}$ to the "head" of $\vec{C}$.

17. It should be mentioned that an efficient way to work this vector addition problem is with the cosine law for general triangles (and since $\vec{a}$, $\vec{b}$ and $\vec{r}$ form an isosceles triangle, the angles are easy to figure). However, in the interest of reinforcing the usual systematic approach to vector addition, we note that the angle $\vec{b}$ makes with the +x axis is 30° + 105° = 135° and apply Eqs.3-5 and 3-6 where appropriate. (**a**) The x component of $\vec{r}$ is $r_x = 10\cos 30° + 10\cos 135° = 1.59$ (m). (**b**) The y component of $\vec{r}$ is $r_y = 10\sin 30° + 10\sin 135° = 12.1$ (m). (**c**) The magnitude of $\vec{r}$ is $(1.59^2 + 12.1^2)^{1/2}$ = 12.2 (m). (**d**) The angle between $\vec{r}$ and the +x direction is $\tan^{-1}(12.1/1.59) = 82.5°$.

31. (**a**) As can be seen from Fig. 3-32, the point diametriccally opposite the origin (0, 0, 0) has position vector $a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$ and this is the vector along the "body diagonal." (**b**) From the point (a, 0, 0) which corresponds to the position vector $a\,\hat{\mathrm{i}}$, the diametrically opposite point is $(0, a, a)$ with the position vector $a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\,\hat{\mathrm{i}} + a\,\hat{\mathrm{j}} + a\,\hat{\mathrm{k}}$. (**c**) If the starting point is (0, a, 0) with the corresponding position vector $a\,\hat{\mathrm{j}}$, the diametrically opposite point is (a, 0, a) with the position vector $a\hat{\mathrm{i}}+a\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $a\hat{\mathrm{i}}-a\hat{\mathrm{j}}+a\hat{\mathrm{k}}$. (**d**) If the starting point is (a, a, 0) with the corresponding position vector $a\hat{\mathrm{i}}+a\hat{\mathrm{j}}$, the diametrically opposite point is (0, 0, a) with the position vector $a\hat{\mathrm{k}}$. Thus, the vector along the line is the difference $-a\hat{\mathrm{i}}-a\hat{\mathrm{j}}+a\hat{\mathrm{k}}$. (**e**) Consider the vector from the back lower left corner to the front upper right corner. It is $a\hat{\mathrm{i}}+a\hat{\mathrm{j}}+a\hat{\mathrm{k}}$. We may think of it as the sum of the vector $a\hat{\mathrm{i}}$ parallel to the x axis and the vector $a\hat{\mathrm{j}}+a\hat{\mathrm{k}}$ perpendicular to the x axis. The tangent of the angle between the vector and the x axis is the perpendicular component divided by the parallel component. Since the magnitude of the perpendicular component is $(a^2+a^2)^{1/2}=\sqrt{2}\,a$ and the magnitude of the parallel component is a, $\tan\theta = a\sqrt{2}/a = \sqrt{2}$. Thus θ = 54.7°. The angle between the vector and each of the other two adjacent sides (the y and z axes) is the same as is the angle between any of the other diagonal vectors and any of the cube sides adjacent to them.



(**f**) The length of any of the diagonals is given by $\sqrt{a^2+a^2+a^2}=\sqrt{3}\,a$.

32. (**a**) With a = 17.0 m and θ = 56.0° we find $a_x = a\cos\theta = 9.51\,\mathrm{m}$. (**b**) And $a_y = a\sin\theta = 14.1\,\mathrm{m}$. (**c**) The angle relative to the new coordinate system is θ' = (56.0°–18.0°) = 38.0°. Thus, $a_x' = a\cos\theta'$ = 13.4 m. (**d**) And $a_y' = a\sin\theta' = 10.5$ m.

40. Using that $\hat{\mathrm{i}}\times\hat{\mathrm{j}}=\hat{\mathrm{k}}$, $\hat{\mathrm{j}}\times\hat{\mathrm{k}}=\hat{\mathrm{i}}$, and $\hat{\mathrm{k}}\times\hat{\mathrm{i}}=\hat{\mathrm{j}}$, we obtain $2\vec{A}\times\vec{B} = 2(2.00\hat{\mathrm{i}}+3.00\hat{\mathrm{j}}-4.00\hat{\mathrm{k}})\times(-3.00\hat{\mathrm{i}}+4.00\hat{\mathrm{j}}+2.00\hat{\mathrm{k}}) = (44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}})$. Next, making use of $\hat{\mathrm{i}}\cdot\hat{\mathrm{i}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{k}}=1$, $\hat{\mathrm{i}}\cdot\hat{\mathrm{j}}=\hat{\mathrm{j}}\cdot\hat{\mathrm{k}}=\hat{\mathrm{k}}\cdot\hat{\mathrm{i}}=0$, we obtain $3\vec{C}\cdot(2\vec{A}\times\vec{B})$

$$= (7.00\hat{\mathrm{i}}-8.00\hat{\mathrm{j}})\cdot(44.0\hat{\mathrm{i}}+16.0\hat{\mathrm{j}}+34.0\hat{\mathrm{k}})$$

$$= 3[(7.00)(44.0)-(8.00)(16.0)] = 540\,.$$

39. Since $ab\cos\phi = a_xb_x + a_yb_y + a_zb_z$,

$$\cos\phi = \frac{a_xb_x + a_yb_y + a_zb_z}{ab}\,.$$

The magnitudes of the vectors given in the problem are $a = |\vec{a}\,| = (3.00^2+3.00^2+3.00^2)^{1/2} = 5.20$,

$$b = |\vec{b}\,| = (2.00^2+1.00^2+3.00^2)^{1/2} = 3.74\,.$$

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

The angle between them is found from $\cos\phi = \dfrac{(3.00)(2.00)+(3.00)(1.00)+(3.00)(3.00)}{(5.20)(3.74)} = 0.926$.

The angle is ϕ = 22°.

55.• The two vectors are given by

$$\vec{A} = 8.00(\cos 130°\,\hat{\mathrm{i}}+\sin 130°\,\hat{\mathrm{j}}) = -5.14\,\hat{\mathrm{i}}+6.13\,\hat{\mathrm{j}}$$

and $\vec{B} = B_x\hat{\mathrm{i}}+B_y\hat{\mathrm{j}} = -7.72\,\hat{\mathrm{i}}-9.20\,\hat{\mathrm{j}}$.

(**a**) The dot product of $5\vec{A}\cdot\vec{B}$ is

$$5\vec{A}\cdot\vec{B} = 5(-5.14\,\hat{\mathrm{i}}+6.13\,\hat{\mathrm{j}})\cdot(-7.72\,\hat{\mathrm{i}}-9.20\,\hat{\mathrm{j}})$$

$$= 5[(-5.14)(-7.72)+(6.13)(-9.20)] = -83.4\,.$$

(**b**) In unit vector notation

$$4\vec{A}\times 3\vec{B} = 12\,\vec{A}\times\vec{B} = 12(-5.14\,\hat{\mathrm{i}}+6.13\,\hat{\mathrm{j}})\times(-7.72\,\hat{\mathrm{i}}-9.20\,\hat{\mathrm{j}}) = 12(94.6\,\hat{\mathrm{k}}) = 1.14\times10^3\,\hat{\mathrm{k}}\,.$$

(**c**) Note that the azimuthal angle is undefined for a vector along the z axis. Thus, our result is "1.14×10^3, θ not defined, and ϕ = 0°". (**d**) Since $\vec{A}$ is in the xy plane, and $\vec{A}\times\vec{B}$ is perpendicular to that plane, then the answer is 90°. (**e**) Clearly, $\vec{A}+3.00\,\hat{\mathrm{k}} = -5.14\hat{\mathrm{i}}+6.13\hat{\mathrm{j}}+3.00\,\hat{\mathrm{k}}$. (**f**) The Pythagorean theorem yields magnitude $A = (5.14^2+6.13^2+3.00^2)^{1/2}$ = 8.54. The azimuthal angle is θ = 130°, just as it was in the problem statement [$\vec{A}$ is the projection onto to the xy plane of the new vector created in part (e)]. The angle measured from the +z axis is $\phi = \cos^{-1}(3.00/8.54) = 69.4°$.

64.• The point P is displaced vertically by $2R$, where R is the radius of the wheel. It is displaced horizontally by half the circumference of the wheel, or πR. Since R = 0.450 m, the horizontal component of the displacement is 1.414 m and the vertical component of the displacement is 0.900 m. If the x axis is horizontal and the y axis is vertical, the vector displacement is $\vec{r} = 1.414\,\mathrm{m}\,\hat{\mathrm{i}}+0.900\,\mathrm{m}\,\hat{\mathrm{j}}$. The displacement has a magnitude of

$$|\vec{r}\,| = \sqrt{(\pi R)^2+(2R)^2} = R\sqrt{\pi^2+4} = 1.68\ \mathrm{(m)},$$

and an angle of

$$\tan^{-1}(2R/\pi R) = \tan^{-1}(2/\pi) = 32.5°,$$

above the floor. In physics there are no "exact" measurements, yet that angle computation seemed to yield something *exact*. However, there has to be some uncertainty in the observation that the wheel rolled half of a revolution, which introduces some indefiniteness in our result.

***Ex.*1-1**, ***Prob.*3-35** & ***Ex.*1-2**, ***Prob.*3-38**.

重點整理－第 3 章　向　量

•**向量**: 須要<u>量值</u>及<u>方向</u>才能描述完整的量; 可用指向性之線段以表示, 線段之兩端點: 起點(箭尾)及終點(箭頭), 線段之<u>長度</u>及<u>指向</u>分別表示向量之<u>大小</u>及<u>方向</u>, 如位移,速度,加速度及力等。

•**向量相加**: 性質相同的物理量才能相加。

•**單位向量**: 大小為 1 之向量,

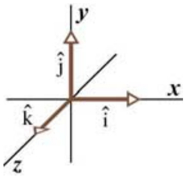
可用以表示方向; 單位向量沒有"單位";

$\hat{i} \parallel x$ axis, $\hat{j} \parallel y$ axis, $\hat{k} \parallel z$ axis,

$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$,

$\hat{i}\cdot\hat{j}=\hat{i}\cdot\hat{k}=\hat{j}\cdot\hat{k}=0$;

$A=|\vec{A}|=(A_x^2+A_y^2+A_z^2)^{1/2}$,



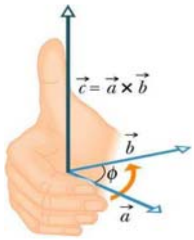
•**內積** $\vec{A}\cdot\vec{B}\equiv AB\cos\theta=A_xB_x+A_yB_y+A_zB_z$;

•**向量積** $\vec{A}\times\vec{B}=C\hat{C}$, $C\equiv AB\sin\theta>0$, where $0\le\theta\le 180°$, and $\vec{C}\perp\vec{A}$ & $\vec{C}\perp\vec{B}$. $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$, $\hat{i}\times\hat{i}=\hat{j}\times\hat{j}=\hat{k}\times\hat{k}=0$;

$\vec{A}\times\vec{B}=(A_yB_z-A_zB_y)\hat{i}+(A_zB_x-A_xB_z)\hat{j}+(A_xB_y-A_yB_x)\hat{k}$;

as $\theta=90° \Rightarrow \vec{A}\cdot\vec{B}=0$;

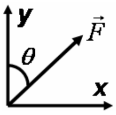
as $\theta=0°$ or $180° \Rightarrow \vec{A}\times\vec{B}=0$.



坐標軸轉動後, 向量的分量會改變, 但其大小仍不變; 物理量會隨坐標系統而改變, 但物理定律仍保持不變

Q. $(2.0\hat{i}-3.0\hat{j}+4.0\hat{k})\times(\hat{i}+\hat{j}+\hat{k})$的向量積為何?
Ans. $-7.0\hat{i}+2.0\hat{j}+5.0\hat{k}$. (↓) *Ans.* (b)

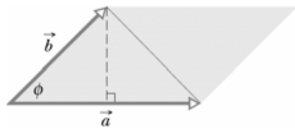
Q. 圖中力向量$\vec{F}$之大小 $F=8.4$ N 及夾角$\theta=60°$，試計算內積$\vec{F}\cdot\vec{j}$(即分量F_y). (a) 8.4 (b) 4.2 N (c) 7.3 N (d) 7.2 N。



How does the ant know the way home with no guiding clues on the desert plain?
螞蟻在沙漠曠野中毫無指引線索下如何知道返巢之路？

scalar,純量; vector (sum),向量(和); resultant,合成量; resolving the vector,分解向量; unit vector,單位向量; component,分量; vector/scalar component,向量/純量分量; component notation,分量記法; magnitude-angle notation,大小-角度記法; coordinated system,座標系統; scalar/dot/inner product,純量/點/內乘積; vector/cross/outer product,向量/叉/外乘積; right-hand rule,右手定則; parallelelogram,平行四邊形; (right) triangle,(直角)三角形; base,底邊,基數,壘; altitude,高; hypotenuse,斜邊; diagonal,對角線(的); displacement,位移; east of north, 往北偏東; commutative/associate/distributive law,交換/結合/分配律; haphazard,無計劃的,隨意的,雜亂的; landmark,地標;work,功; torque,力矩;

S1. Show that the area of the triangle contained between $\vec{a}$ and $\vec{b}$ and the solid line in the figure below is $\frac{1}{2}|\vec{a}\times\vec{b}|$.



Sol. The area of a triangle is half the product of its base and altitude. The base is the side formed by vector $\vec{a}$. Then the altitude is $b\sin\phi$ and the area is

$$A=\frac{1}{2}ab\sin\phi=\frac{1}{2}|\vec{a}\times\vec{b}|.$$

為由$\vec{a}$及$\vec{b}$相鄰邊組成之平行四邊形的面積之半

•"**新一代 GPS**"，艾胥利，科學人 2003 年 10 月。 • "**GPS：讓路痴不再迷路**"，哈奇森，科學人 2004 年 6 月。 • "**GPS：讓飛航更安全、更準時**"，翁千婷，科學人 2004 年 6 月。 •**備忘錄**•