77. Assuming the horizontal velocity of the ball is *constant*, the horizontal displacement is $\Delta x = v\Delta t$, where Δx is the horizontal distance traveled, Δt is the time, and v is the (horizontal) velocity. With v = 160 km/h = 44.4 m/s, we have

$$\Delta t = \frac{\Delta x}{\Delta t} = \frac{18.4}{44.4} = 0.414 \text{ (s)}.$$

02. Average speed, as opposed to average velocity, relates to the total distance, as opposed to the net displacement. The distance D *up* the hill is, of course, the same as the distance *down* the hill, and since the speed is constant (during each stage of the motion) we have speed = D/t. Thus, the average speed is

$$\frac{D_{up} + D_{down}}{t_{up} + t_{down}} = \frac{2D}{D/v_{up} + D/v_{down}},$$

which, after canceling D and plugging in v_{up} = 40 km/h and v_{down} = 60 km/h, yields 48 km/h for the average speed.

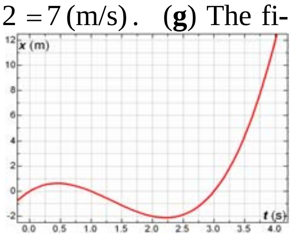
04. With 1 m/s = 3.6 km/h, Huber's speed is

$$v_0 = (200\text{ m})/(6.509\text{ s}) = 30.72\text{ m/s} = 110.6\text{ km/h}.$$

Since Whittingham beat Huber by 19.0 km/h, his speed is $v_1 = 110.6 + 19.0 = 129.6$ (km/h), or 36.00 m/s. Thus, the time through a distance of 200 m for Whittingham is

$$\Delta t = \Delta x / v_1 = (200\text{ m})/(36.00\text{ m/s}) = 5.554\text{ s}.$$

05. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write $x = (3\text{ m/s})t - (4\text{ m/s}^2)t^2 + (1\text{ m/s}^3)t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules *rigorously*. (**a**) Plugging in $t = 1$ s yields $x = 3 - 4 + 1 = 0$. (**b**) With t = 2 s we obtain $x = 3(2) - 4(2)^2 + (2)^3 = -2$ (m). (**c**) With $t = 3$ s we have $x = 0$ m. (**d**) Plugging in $t = 4$ s gives x = 12 m. For later reference, we also note that the position at t = 0 is x = 0. (**e**) The position at t = 0 is subtracted from the position at t = 4 s to find the displacement $x = 12$ m. (**f**) The position at $t = 2$ s is subtracted from the position at t = 4 s to give the displacement x = 14 m. Eq.2-2, then, leads to $v_{av} = \Delta x/\Delta t = 14/2 = 7$ (m/s). (**g**) The figure is shown for horizontal axis of $0 \le t \le 4$ s. Draw a straight line from the point at (2, –2) to that at (4, 12), whose slope give the answer for part (f).



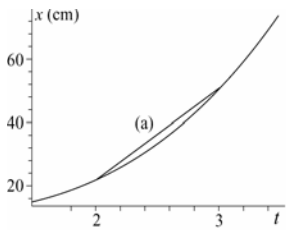
17. We use Eq.2-2 for average velocity and Eq.2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds. (**a**) We plug into the given equation for x for t = 2.00 s and t = 3.00 s and obtain x_2 = 21.75 cm and x_3 = 50.25 cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{50.25 - 21.75}{3.00 - 2.00},$$

which yields v_{av} = 28.5 cm/s. (**b**) The instantaneous velocity is $v = dx/dt = 4.5t^2$, which, at time t = 2.00 s, yields $v = (4.5)(2.00)^2 = 18.0$ (cm/s). (**c**) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2$ = 40.5 (cm/s). (**d**) At t = 2.50 s, the instanttaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ (cm/s). (**e**) Let t_m stand for the moment when the particle is midway between x_2 and x_3 [that is, when the particle is at $x_m = (x_2 + x_3)/2 = 36$ cm]. Therefore,

$$x_m = 9.75 + 1.5t_m^3 \quad \Rightarrow \quad t_m = 2.596\text{ s}.$$

Thus, the instantaneous speed at this time is $v = 4.5\times(2.596)^2 = 30.3$ (cm/s). (**f**) The answer to part (a) is given by the slope of the straight line between t = 2 & t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (*not* shown but easily imagined) to the curve at the appropriate points.



30. The acceleration found from Eq.2-11 (or, suitably interpreted, Eq.2-7) is $a = \Delta v / \Delta t$ = (1020 km/h)/(1.4 s) = (1020 m)/(3.6×1.4 s^2) = 202.4 m/s^2. In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = (202.4/9.8)\,g = 20.6\,g = 21\,g.$$

31. We assume the periods of acceleration (duration t_1) and deceleration (duration t_2) are periods of constant a so that Table 2-1 can be used. Taking the direction of motion to be $+x$ then $a_1 = +1.22\text{ m/s}^2$ and $a_2 = -1.22\text{ m/s}^2$. We use SI units so the velocity at $t = t_1$ is $v = 305/60 = 5.08$ (m/s). (**a**) We denote Δx as the distance moved during t_1, and use Eq. 2-16:

$$v^2 = v_0^2 + 2a_1\Delta x$$
$$\Rightarrow \Delta x = 5.08^2/[2(1.22)] = 10.59 \approx 10.6\text{ (m)}.$$

(**b**) Using Eq. 2-11, we have

$$t_1 = (v - v_0)/a_1 = 5.08/1.22 = 4.17\text{ (s)}.$$

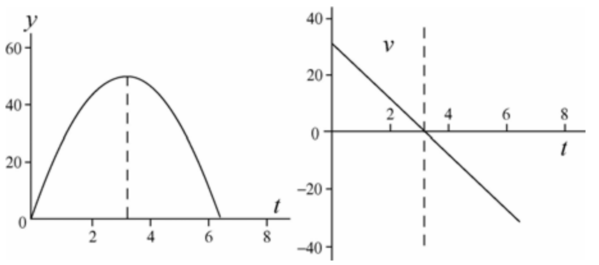
The deceleration time t_2 turns out to be the same so that $t_1 + t_2$ = 8.33 s. The distances traveled during t_1 and t_2 are the same so that they total to 2(10.59) = 21.18 (m). This implies that for a distance of 190 – 21.18 = 168.82 (m), the elevator is traveling at constant velocity. This time of constant velocity motion is $t_3 = 168.82/5.08 = 33.21$ (s). Therefore, the total time is 8.33 + 33.21 = 41.5 (s).

47. We neglect air resistance for the duration of the motion (between "launching" and "landing"), so $a = -g$

$= -9.8\,\mathrm{m/s^2}$ (we take downward to be the $-y$ direction). We use the equations in Table 2-1 (with Δy replacing Δx) because this is a = constant motion. (**a**) At the highest point the velocity of the ball vanishes. Taking $y_0 = 0$, we set $v = 0$ in $v^2 = v_0^2 - 2gy$ and solve for the initial velocity: $v_0 = (2gy)^{1/2}$. Since $y = 50\,\mathrm{m}$ we find $v_0 = 31\,\mathrm{m/s}$. (**b**) It will be in the air from the time it leaves the ground until the time it returns to the ground ($y = 0$). Applying Eq. 2-15 to the entire motion (the rise and the fall, of total time $t > 0$) we have

$$y = 0 = v_0 t - \tfrac{1}{2} g t^2 \quad \Rightarrow \quad t = 2v_0/g\,,$$

which [using our result from part (a)] produces t = 6.4 s. It is possible to obtain this without using part (a)'s result; one can find the time just for the rise (from ground to highest point) from Eq.2-16 and then double it. (**c**)* SI units are understood in the y and v graphs shown below. In the interest of saving space, we do not show the graph of a, which is a horizontal line at $-9.8\,\mathrm{m/s^2}$.



56. (**a**) We use primed variables (except t) with the first stone, which has zero initial velocity, and unprimed variables with the second stone (with initial *downward* velocity $-v_0$, so that v_0 is being used for the *initial* speed). SI units are used throughout.

$$\Delta y' = 0(t) - \tfrac{1}{2} g t^2, \quad \Delta y = (-v_0)(t-1) - \tfrac{1}{2} g(t-1)^2\,.$$

Since the problem indicates $\Delta y' = \Delta y = -43.9\,\mathrm{m}$, we solve the first equation for t (finding $t = 2.99\,\mathrm{s}$) and use this result to solve the second equation for the initial speed of the second stone:

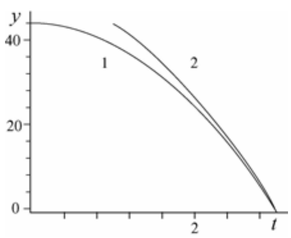
$$-4.39 = (-v_0)(2.99-1) - (1/2)(9.80)(2.99-1)^2,$$

which leads to $v_0 = 12.3\,\mathrm{m/s}$.

(**b**)* The velocity of the stones are given by

$$v_y' = d(\Delta y')/dt = -gt,$$
$$v_y = d(\Delta y)/dt = -v_0 - g(t-1)\,.$$

The plot is shown right:



50. The full extent of the bolt's fall is given by $y - y_o = -(½)gt^2$ where $y - y_o = -90\,\mathrm{m}$ (if *upwards* is chosen as the $+y$ direction). Thus the time for the full fall is found to be $t = 4.29\,\mathrm{s}$. The first 80% of its free fall distance is given by $-72 = -gt_{80}^2/2$, which requires time $t_{80} = 3.83\,\mathrm{s}$. (**a**) Thus, the final 20% of its fall takes $t - t_{80} = 0.45\,\mathrm{s}$. (**b**) We can find that speed using $v = -gt_{80}$. Therefore, $|v| = 38\,\mathrm{m/s}$, approximately. (**c**) Similarly, $v_{final} = -gt_{80} \Rightarrow |v_{final}| = 42\,\mathrm{m/s}$.

99.• We neglect air resistance, which justifies setting $a = -g = -9.8\,\mathrm{m/s^2}$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is *constant* acceleration motion. When something is thrown straight *up* and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-25), the time of flight t is half of its time of ascent t_a, which is given by Eq.2-18 with $\Delta y = H$ and $v = 0$ (indicating the maximum point).

$$H = v\,t_a + \tfrac{1}{2} g\,t_a^2 \quad \Rightarrow \quad t_a = \sqrt{2H/g}\,.$$

Writing these in terms of the total time in the air $t = 2t_a$ we have $H = \tfrac{1}{8} g t^2 \Rightarrow t = 2\sqrt{2H/g}\,.$

We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio: $\dfrac{H_2}{H_1} = \dfrac{g t_2^2/8}{g t_1^2/8} = (\dfrac{t_2}{t_1})^2$

from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2 H_1 = 4H_1$.

101.• Taking the $+y$ direction *downward* and $y_0 = 0$, we have $y = v_0 t + (½)gt^2$. which (with $v_0 = 0$) yields $t = (2y/g)^{1/2}$. (**a**) For this part of the motion, $y = 50$ m so that $t = \sqrt{2(50)/9.8} = 3.2\,(\mathrm{s})$.

(**b**) For this next part of the motion, we note that the total displacement is $y = 100\,\mathrm{m}$. Therefore, the total time is $t = \sqrt{2(100)/9.8} = 4.5\,(\mathrm{s})$.

The different between this and the answer to part (a) is the time required to fall through that second 50 m distance: 4.5 – 3.2 = 1.3 (s).

***Ex.*2-1**, ***Prob.*2-71** & ***Ex.*2-2**, ***Prob.*2-88**.

•高速鐵路是指行車時速達二百公里以上的行車系統(電車), 台灣高鐵((960105 營運通車)之新幹線電車平均行車速率 230 km/h (63.9 m/s), 最高時速 300 km/h, 台北至高雄左營距 345 km, 行車需 90 分.

•世上最快電車—法國 TGV 最高時速可達 574.8 km (357.2 英里), 而上海磁浮車時速達 435 km.

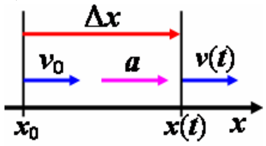
•**頸骨受傷**/鞭樣損傷：是車禍常見的症狀，主要因為車禍發生時，頭部及頸部的脊骨就如鞭子鞭動一樣，因突然而來的劇動而受傷。•**頭部受傷規範**(HIC $= \underline{a}^{2.5}$, $\underline{a}$: 頭部減速度大小) 當 HIC > 1 千: 受傷機率 15%, 1 千 k ~ 3 千, 嚴重骨折及腦部傷害.

J1.“交通與物理”，黃定維及黃偉能，物理雙月刊，24 卷 2 期(2004 年 4 月) 334-337。J2. “***The evolution of transport***”, J. H. Ausubel and C. Marchetti, *The Industrial Physicist*, April/May 2001, pp. 20-24。

重點整理－第 2 章　直線運動

宇宙萬物皆在動，但如何動？力學包含**運動學**與**動力學**；運動學是力學之初步。**運動學**旨在探討物體如何運動，即描述物體在空間的位置 $\mathbf{r}$ 與時間 t 之關係, $\mathbf{r}=\mathbf{r}(t)$；只用“**空間**”及“**時間**”兩基本概念，無“**力**”與“**質量**”等概念。**動力學**: 探討運動之起因，物體為什麼作這樣運動。即研討力、質點(或物體)以及運動物體之間的關係，或物體運動時所遵守的定律或法則－**牛頓運動定律**。

一維或直線運動 $x=x(t)$,



時間 $t_1\to 0 \Rightarrow t_2\to t$,

時距 $\Delta t\equiv t_2-t_1\to t$,

初位置 $x_1\equiv x(t_1)\to x_0\Rightarrow$ **末位置** $x_2\equiv x(t_2)\to x$;

位移為“位置之改變量” $\Delta x\equiv x_2-x_1\to x-x_0$,

只與**初位置**及**末位置**有關，但與運動細節無關.

平均速度為“單位時間之位移” $v_{av}\equiv \Delta x/\Delta t$,

(瞬時)**速度**為位置之時變率 $v=dx/dt$,

速率指(a)速度大小或(b)運動總路程除以時距

初速度 $v_1\equiv v(t_1)\to v_0\Rightarrow$末速度 $v_2\equiv v(t_2)\to v$;

速度改變量$\Delta v\equiv v_2-v_1\to v-v_0$,

平均加速度為單位時間之速度改變量 $a_{av}\equiv \Delta v/\Delta t$,

(瞬時)**加速度**為速度之時變率 $a=dv/dt=d^2x/dt^2$,

一維等加速度運動 $a=a_{av}=$ const.,

$v_{av}=\frac{1}{2}(v_0+v)$, 即‘**初速度**’與‘**末速度**’之平均值,

$\Delta v=a\Delta t=at$ or $v=v_0+at$,

$\Delta x=v_{av}\Delta t=v_0t+\frac{1}{2}at^2$, or $x=x_0+v_0t+\frac{1}{2}at^2$,

$v^2-v_0^2=2a\Delta x$, or $v^2=v_0^2+2a(x-x_0)$,

Note $v=0$ 表示運動停止或運動方向即將改變(即前後速度變號), 該點為**折返點**;

自由下落運動：設鉛直方向之加速度為定值

$x\to y$ and $a\to a_y=-g$ ($\Delta y>0$: up),

地表自由下落加速度 $g=9.8\,\text{m/s}^2=49/5\,\text{m/s}^2$.

•人類跑百米最快需 9.69 s (2009 柏林,牙買加柏特)

•1971 (美)阿波羅**十五號**太空人在月球表面，左手持**鐵錘**，右手拿**羽毛**，同時釋放，結果鐵錘與羽毛同時著地。•地震時地表加速度以 gal 表示, $1\,\text{gal}=1\,\text{cm/s}^2$.

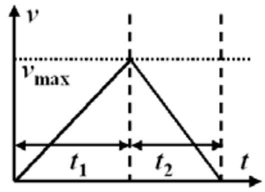
•基隆之 $g=9.78974\,\text{m/s}^2$。

Why can a woodpecker survive the severe impacts with a tree limb?

為何啄木鳥激烈的撞擊大樹枝還能存活？

解題策略: **1.問題**瞭解嘛？**2.單位**正確嘛？**3.答案**合理嘛？**4.讀懂圖形**。 *Note* $d(x^n)/dx=nx^{n-1}$.

挑戰題〉設直線賽車比賽總長為 S, 若跑車前段加速 ($a_1>0$),而後段減速($-a_2<0$), 試計算跑車所花時間。



Sol. Let the distance during the period t_1 of the acceleration a_1 be S_1 and the distance during the period t_2 of the acceleration $-a_2$ be S_2. The maximum speed of car is

$v_{max}^2=2a_1S_1=2a_2S_2$. From $S=S_1+S_2$, we have

$S_1=\frac{a_2S}{a_1+a_2}=\frac{1}{2}a_1t_1^2$ and $S_2=\frac{a_1S}{a_1+a_2}=\frac{1}{2}a_2t_2^2$.

Solve them to find $t_1=[\frac{2a_2S}{a_1(a_1+a_2)}]^{1/2}$ and

$t_2=[\frac{2a_1S}{a_2(a_1+a_2)}]^{1/2}$, so $\Delta t=t_1+t_2=[\frac{2(a_1+a_2)S}{a_1a_2}]^{1/2}$.

Using $S=0.25$ mi $=1320$ ft, $a_1=24\,\text{ft/s}^2$, and $a_2=-32\,\text{ft/s}^2$ leads to $\Delta t=13.9$ s. *cf. Prob.2-29.*

例.某車速 180 km/h 之車子突然煞車, 若煞車距離為 175 m, 試計算煞車過程之加速度. (*Ans.* $-7.14\,\text{m/s}^2$)

例.某球以 19.6 m/s 速率從地面往上拋, 試計算該球可抵達之最大高度(a)及於空中停留時間(b).

motion,運動; kinematics,運動學; particle,質點; position 位置; origin/zero point,原點; positive/negative direction,正/負的方向; axis (座標)軸; coordinate,座標; vector,向量; displacement,位移; distance,距離; total distance,總距離/路程; time interval,時距; travel,行進; slope,斜率; average velocity,平均速度; (instantaneous) velocity,(瞬時)速度; speed,速率; speedometer,速率錶; average acceleration,平均加速度; (instantaneous) acceleration (瞬時)加速度; constant acceleration,等加速度, free-fall,自由下落; derivative,導數/微商; tectonic plate,板塊; artery,動脈; whiplash injury,頸部扭傷; head restraint,頭枕; woodpecker,啄木鳥; beak,鳥啄; rat-tat,咚咚的聲音,敲擊聲; fanatical,狂熱的/入迷的; armadillo,犰狳(中南美產); beat-up,用壞了的; pickup 臨時湊成的/偶然認識的; sprinter,短跑選手; energy conservation (*EC*); Niagara Falls,尼加拉瓜瀑布; Porche,保時捷; NASCAR: National Association of Stock Car Auto Racing,全國運動汽車競賽協會; jai alai,回力球(類似手球的室內遊戲,盛行於拉丁美洲) •1 knot = 1.852 km/h •**備忘錄**•