Chapter **30** *Induction and Inductance*

04. The resistance of the loop is $R = \rho L / A = (1.69\times10^{-8})\pi(0.10) / [4\pi(2.5\times10^{-3})^2] = 1.1\times10^{-2}$ (Ω). We use $i = |\varepsilon| / R = |d\Phi_B/dt| / R = (\pi r^2/R)|dB/dt|$. Thus $dB/dt = iR / \pi r^2$

$= (10)(1.1\times10^{-2}) / \pi (0.05)^2 = 1.4$ (T/s).

06. Using Faraday's law, the induced emf is

$\varepsilon = -d\Phi_B/dt = -d(BA) / dt = -BdA/dt = -Bd(\pi r^2)/dt = -2\pi rBdr/dt = -2\pi(0.12)(0.800)(-0.750) = 0.452$ (V).

08. The field (due to the current in the straight wire) is out-of-the-page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

16. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero compo- nent perpendicular) to the coil, and must be changing with time. (**a**) For $\boldsymbol{B} = (4.00\times10^{-2}$ T/m)$y\mathbf{k}$, $dB/dt = 0$ and hence $\varepsilon = 0$. (**b**) None. (**c**) For $\boldsymbol{B} = (6.00\times10^{-2}$ T/s)$t\mathbf{k}$, $\varepsilon = -d\Phi_B/dt = -AdB/dt = (0.400 \times 0.250)(0.0600) = -6.00$ (mV), or $|\varepsilon| = 6.00$ mV. (**d**) Clockwise; (**e**) For $\boldsymbol{B} = (8.00\times10^{-2}$ T/s)$yt\,\mathbf{k}$,

$$\Phi_B = (0.400)(0.0800t)\int ydy = 1.00\times10^{-3}t,$$

in SI units. The induced emf is $\varepsilon = -d\Phi_B /dt = -1.00$ (mV), or $|\varepsilon| = 1.00$ mV. (**f**) Clockwise. (**g**) $\Phi_B = 0 \Rightarrow \varepsilon = 0$. (**h**) None. (**i**) $\Phi_B = 0 \Rightarrow \varepsilon = 0$. (**j**) None.

19. (**a**) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. *Eq*. 29-27, with $z = x$ (taken to be much greater than R), gives

$$\boldsymbol{B} = (\mu_0 iR^2 / 2x^3)\mathbf{i},$$

where the $+x$ direction is upward in Fig. 30-47. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = (\mu_0 iR^2 / 2x^3)(\pi r^2) = \pi\mu_0 i\, r^2R^2/2x^3.$$

(**b**) The emf is given by Faraday's law:

$$\varepsilon = -d\Phi_B / dt = -(\pi\mu_0 i\, r^2R^2 / 2)d(x^{-3})/dt$$
$$= (\pi\mu_0 i\, r^2R^2 / 2)(3x^{-4})dx/dt = 3\pi\mu_0 i\, r^2R^2v / 2x^4.$$

(**c**) As the smaller loop moves upward, the flux through it decreases, and we have a situation like that shown in Fig. 30-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

24. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by *Eq*. 29-17). We integrate according to *Eq*. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2}(\mu_0 i/2\pi r)(adr)$$
$$= (\mu_0 ia/2\pi)\ln[(r+b/2)/(r-b/2)].$$

When $r = 1.5b$, we have

$$\Phi_B = (2\times10^{-7})(4.7)(0.022)\ln(2.0) = 1.4\times10^{-8} \text{ (Wb)}.$$

(**b**) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $dr/dt = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$i_{\text{loop}} = |\varepsilon / R| = (\mu_0 ia/2\pi)(d/dt)\ln[(r+b/2)/(r-b/2)]$$
$$= \mu_0 iabv / 2\pi R(r^2-b^2/4) = (2\times10^{-7})(4.7)(0.022)$$
$$(0.0080)(3.2\times10^{-3}) / (4.0\times10^{-4}) / (2\times0.0080^2)$$
$$= 1.5\times10^{-5} \text{ (A)}.$$

26. Noting that $|\Delta B| = B$, we find the thermal energy is $P_{\text{thermal}}\Delta t = \varepsilon^2\Delta t / R = R^{-1}(-d\Phi_B/dt)^2\Delta t$

$$= R^{-1}(-A\Delta B/\Delta t)^2\Delta t = A^2B^2 / R\Delta t =$$
$$= (2.00\times10^{-4})^2(17.0\times10^{-6})^2/(5.21\times10^{-6})$$
$$/(2.96\times10^{-3}) = 7.50\times10^{-10} \text{ (J)}.$$

29. (**a**) *Eq*. 30-8 leads to

$$\varepsilon = BLv = (0.350)(0.250)(0.550) = 0.0481 \text{ (V)}.$$

(**b**) By Ohm's law, the induced current is $i = 0.0481$ V/18.0 Ω = 0.00267 A. By Lenz's law, the current is clockwise in Fig. 30-52.

(**c**) *Eq*. 26-22 leads to $P = i^2R = 0.000129$ W.

39. Since $N\Phi_B = Li$, we obtain $\Phi_B = Li / N = (8.0\times10^{-3})(5.0\times10^{-3}) / 400 = 1.0\times10^{-7}$ (Wb).

43. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting $di/dt = -\varepsilon / L = -(60$ V$)$ / (12 H) = –5.0 A/s, or $|di/dt| = 5.0$ A/s. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

48. The steady state value of the current is also its maximum value, ε/R, which we denote as i_m. We are told that $i = i_m/3$ at $t_0 = 5.00$ s. *Eq*. 30-41 becomes $i = i_m[1-\exp(-t_0/\tau_L)]$ which leads to

$$\tau_L = -t_0 / \ln(1-i/i_m) = -5.00 \text{ s} / \ln(2/3) = 12.3 \text{ s}.$$

52. (**a**) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \varepsilon / (R_1+R_2) = 100 / (10.0+20.0) = 3.33 \text{ (A)}.$$

(**b**) $i_2 = i_1 = 3.33$ A. (**c**) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1R_1 - i_2R_2 = 0 \text{ and } \varepsilon - i_1 R_1 -(i_1-i_2)R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find $i_1 = \varepsilon(R_2+R_3)/(R_1R_2+R_2R_3+R_3R_1) = (100)(10.0+20.0)$ /(10.0×20.0+20.0×30.0+30.0×10.0) = 4.55 (A).

(**d**) and

$i_2 = \varepsilon R_3/(R_1R_2+R_2R_3+R_3R_1) = (100)(30.0) / (10.0\times20.0+20.0\times30.0+30.0\times10.0) = 2.73$ (A).

(**e**) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$). (**f**) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55A – 2.73A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, i.e., $i_2 = -1.82$ A. A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus, (**g**) $i_1 = 0$, and (**h**) $i_2 = 0$.

63. (**a**) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 ni$, where n, for the solenoid of this *Pb*., is (950 turns)/(0.850 m) = 1.118×10^3 m^{-1}. The magnetic energy density is $u_B = (½)\mu_0 n^2 i^2 = (½)(4\pi\times10^{-7})(1.118\times10^3)^2(6.60)^2 = 3.42$ (J/m^3).

(**b**) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B V_s$, where V_s is the volume of the solenoid. V_s is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = (3.42)(17.0\times10^{-4})(0.850) = 4.94\times10^{-2}\ \text{(J)}.$$

66. (a) The magnitude of the magnetic field at the center of the loop, using *Eq*. 29-9, is

$$B = \mu_0 i/2R = (4\pi\times10^{-7})(100) / (2\times50\times10^{-3}) = 1.3\times10^{-3}\ \text{T}.$$

(**b**) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = B^2/2\mu_0 = (1.3\times10^{-3})^2 / (2\times4\pi\times10^{-7}) = 0.63\ \text{(J/m}^3\text{)}.$$

45. (**a**) Voltage is proportional to inductance (by *Eq.* 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add (V_1+V_2), then inductances in series must *add*, $L_{eq} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other. (**b**) Just as with resistors, $L_{eq} = \Sigma_k L_k$.

73. The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{wire}\, dr = \int_a^{a+b} \frac{\mu i}{2\pi r}\ell dr = \frac{\mu_0 i\ell}{2\pi}\ln\frac{a+b}{a}.$$

Thus, $$M = \frac{N\Phi}{i} = \frac{\mu_0 N\ell}{2\pi}\ln\frac{a+b}{a}.$$

With $N = 100$, $a = 1.0$ cm, $b = 8.0$ cm and $\ell = 30$ cm, we have $M = 1.3\times10^{-5}$ H.

重點整理－第 30 章 感應與電感

磁通量 磁場與面積向量之純量乘積，正比於通過截面之磁場線數；單位: 韋伯 Wb (≡ T·m^2)；

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \int BdA\cos\theta;$$

$$\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta\ (\text{均勻磁場})。$$

Note ***a***.通過封閉曲面之磁通量必為零；***b***.磁場可由磁通量除以截面積得之, 即$[B]$ = T = Wb/m^2。***c***.磁通量計算類似電通量之計算。

法拉第感應定律 $\varepsilon = -\dfrac{d\phi_B}{dt}$, 沿著任意封閉路徑之感應電動勢等於通過此路徑所包圍截面之磁通量時變率之負值。改變 **1**.磁場強度; **2**.線圈面積; **3**.磁場與面積向量之夾角皆可改變磁通量。

冷次定律 感應電動勢(電流)之效應乃是阻止產生此磁通量之改變。此定律易於判斷感應電流之方向或感應電動勢之極性。

電動勢與感應電場 $\varepsilon = \oint \vec{E}\cdot d\vec{\ell} = -\dfrac{d\phi_B}{dt}$.

Note 通常感應電場之封閉路徑線積分值不為零, 但靜電場的封閉路徑線積分值必為零。

電感器 具有特定電感值之元件，電路符號

電感為 $L = \dfrac{N\phi_B}{i}$, 單位: H (≡ T·m^2/A)。

螺線管 $L = \mu_0 n^2 \ell A$, n:線圈密度,A:截面積, ℓ:長度。

自感應 線圈自身電流隨時變化而於其上誘發一電動勢之現象, $\varepsilon_L = -L\dfrac{di}{dt}$.

RL* 串聯電路** 電感時間常數$\tau_L \equiv L/R$；a***.建立電流時，$i(t) = (\varepsilon/R)[1-\exp(-t/\tau_L)]$，***b***.電流衰減時，$i(t) = i_0\exp(-t/\tau_L)$。*Note* ***a***.順著電流跨過電感器, $\Delta V = +\varepsilon_L = -Ldi/dt$; ***b***.電流穩定時, 電感器短路($\varepsilon_L = 0$, $di/dt = 0$), ***c***.電流剛流通時, 電感器斷路($i = 0$, $R_L \to \infty$).

磁能及磁能密度 $U_B = \dfrac{1}{2}Li^2$, $u_B = \dfrac{1}{2\mu_0}B^2$.

互感應 線圈 1 電流隨時變化而於其旁線圈 2 上誘發一電動勢之現象，M (= $M_{12} = M_{21}$)為互電感，

$$\varepsilon_2 = -M\frac{di_1}{dt},\ \varepsilon_1 = -M\frac{di_2}{dt}.\ M\ 單位:\ \text{H}.$$

磁場如何引起燒傷？

induction 感應; Faraday's law of induction 法拉第感應定律; Lanz's law 冷次定律; induced current/emf/electric field 感應電流/電動勢/電場; eddy current 渦電流; magnetic flux 磁通量; magnetic flux linkage 磁通匝連數; weber (Wb)韋伯; inductance 電感; henry (H)亨利; self-/mutual induction 自/互感應; inductor 電感器; MRI 磁振造影;burn 燃燒,燒傷;pickup 拾音器

46. (**a**) Voltage is proportional to inductance (by *Eq*. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the *Eq*. 27-21 for resistors. We note that this condition on the currents implies

$$di_1(t)/dt + di_2(t)/dt = di(t)/dt.$$

Thus, although the inductance equation, *Eq*. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also applies to inductors.

Therefore, $L_{eq}^{-1} = L_1^{-1} + L_2^{-1}$.

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that the field of one inductor should not have significant influence (or "coupling'') in the next. (**b**) Just as with resistors, $L_{eq}^{-1} = \Sigma_k L_k^{-1}$.

Ex.5-2: Pb.30-42.

•**備忘錄**•