Chapter **29** *Magnetic Fields Due to Currents*

02. The straight segment of the wire produces no magnetic field at C (see the *straight sections* discussion in *S.P.* 29-1). Also, the magnetic fields from the two semi-circular loops cancel at C (by symmetry). Therefore, $B_C = 0$.

55. (**a**) We find the magnetic field by superposing the results of two semi-infinite wires (*Eq.* 29-7) and a semi-circular arc (*Eq.* 29-9 with $\phi = \pi$). The direction of $\boldsymbol{B}$ is *out of* the page, as can be checked by referring to Fig. 29-6(c). The magnitude of $\boldsymbol{B}$ at point a is therefore

$$B_a = 2\frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i\pi}{4\pi R} = \frac{\mu_0 i}{2R}\left(\frac{1}{\pi} + \frac{1}{2}\right).$$

With i = 10 A and R = 0.0050 m, $B_a = 1.0\times10^{-3}$ T. (**b**) The direction of this field is *out of* the page, as Fig. 29-6(c) makes clear. (**c**) The last remark in the *Pb.* statement implies that treating b as a point midway bet. two infinite wires is a good approximation. Thus, using Eq. 29-4, $B_b = 2(\mu_0 i/2\pi R) = 8.0\times10^{-4}$ T. (**d**) This field, too, points out of the page.

59. Using the right-hand rule (and symmetry), we see that $\boldsymbol{B}_{\rm net}$ points along what we will refer to as the y axis (passing through P), consisting of two equal magnetic field y-components. Using *Eq.* 29-17, $|\boldsymbol{B}_{\rm net}| = 2(\mu_0 i/2\pi r)\sin\theta$, where i = 4.00 A, $r = (d_1^2 + d_2^2/4)^{-1/2}$ = 5.00 m, and $\theta = \tan^{-1}(2d_2/d_1) = 53.1°$. Therefore, $|\boldsymbol{B}_{\rm net}| = 2.56\times10^{-7}$ T.

06. (**a**) Recalling the *straight sections* discussion in *S.P.* 29-1, we see that the current in the straight segments collinear with C do not contribute to the field at that point. *Eq.* 29-9 (with $\phi = \pi$) indicates that the current in the semicircular arc contributes $\mu_0 i/4R$ to the field at C. Thus, the magnitude of the magnetic field is $B = \mu_0 i/4R = (4\pi\times10^{-7})(0.0348)/(4\times0.0926) = 1.18\times10^{-7}$ (T). (**b**) The right-hand rule shows that this field is into the page.

08. (**a**) Since they carry current in the same direction, then (by the right-hand rule) the only region in which their fields might cancel is between them. Thus, if the point at which we are evaluating their field is r away from the wire carrying current i and is d–r away from the wire carrying current 3.00i, then the canceling of their fields leads to

$$\frac{\mu_0 i}{2\pi r} = \frac{\mu_0 (3i)}{2\pi (d\text{-}r)} \Rightarrow r = \frac{d}{4} = \frac{16.0\,\text{cm}}{4} = 4.0\ \text{cm}.$$

(**b**) Doubling the currents does not change the location where the magnetic field is zero.

21. (**a**) The contribution to B_C from the (infinite) straight segment of the wire is $B_{C1} = \mu_0 i/2\pi R$. The contribution from the circular loop is $B_{C2} = \mu_0 i/2R$. Thus,

$$B = B_{C1} + B_{C2} = (\mu_0 i/2R)(1+\pi^{-1}) = 2.53\times10^{-7}\ \text{T}.$$

$\boldsymbol{B}_C$ points out of the page, or in the $+z$ direction. In unit-vector notation, $\boldsymbol{B}_C = (2.53\times10^{-7}\ \text{T})\mathbf{k}$.
(**b**) Now $\boldsymbol{B}_{C1}\perp\boldsymbol{B}_{C2}$, so $B_C = (B_{C1}^2+B_{C2}^2)^{1/2} = (\mu_0 i/2R)(1+\pi^{-2})^{1/2} = 2.02\times10^{-7}$ T,

and $\boldsymbol{B}_C$ points at an angle θ (relative to the plane of the paper) equal to $\tan^{-1}(B_{C1}/B_{C2}) = \tan^{-1}(1/\pi) = 17.66°$. In unit-vector notation,

$$\boldsymbol{B}_C = B_C(\cos\theta\,\mathbf{i}+\sin\theta\,\mathbf{k}) = (1.92\times10^{-7}\ \text{T})\,\mathbf{I} + (6.12\times10^{-8}\ \text{T})\,\mathbf{k}$$

33. The magnitudes of the forces on the sides of the rectangle which are parallel to the long straight wire (with i_1 = 30.0 A) are computed using *Eq.* 29-13, but the force on each of the sides lying perpendicular to it (along our y axis, with the origin at the top wire and $+y$ downward) would be figured by integrating as follows:

$$F_{\perp\rm sides} = \int_a^{a+b} i_2\frac{\mu_0 i_1}{2\pi\,y}dy\ .$$

Fortunately, these forces on the two perpendicular sides of length b cancel out. For the remaining two (parallel) sides of length L, we obtain

$$F = (\mu_0 i_1 i_2 L/2\pi)[a^{-1}-(a+b)^{-1}] = \mu_0 i_1 i_2 Lb/2\pi a(a+b) = 3.2\times10^{-3}\ \text{N}.$$

and $\boldsymbol{F}$ points toward the wire, or $\mathbf{j}$. In unit-vector notation, we have $\boldsymbol{F} = (3.20\times10^{-3}\ \text{N})\mathbf{j}$.

40. It is possible (though tedious) to use Eq. 29-26 and evaluate the contributions (with the intent to sum them) of all 1200 loops to the field at, say, the center of the solenoid. This would make use of all the information given in the problem statement, but this is not the method that the student is expected to use here. Instead, *Eq.*29-23 for the ideal solenoid (which does not make use of the coil radius) is the preferred method: $B = \mu_0 ni = \mu_0(N/\ell)i$, where i = 3.60 A, ℓ = 0.950 m and N = 1200. This yields B = 0.00571 T.

47. The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. We use $A = \pi R^2$, where R is the radius. Thus,

$$\mu = (200)(0.29)\pi(0.10/2)^2 = 0.46\ (\text{A}\cdot\text{m}^2).$$

50. We use *Eq.* 29-26 and note that the contributions to $\boldsymbol{B}_p$ from the two coils are the same. Thus,

$$B_p = \frac{2\mu_0 iR^2 N}{2[R^2+(R/2)^2]^{3/2}} = \left(\frac{4}{5}\right)^{3/2}\mu_0\frac{Ni}{R} = 8.78\times10^{-6}\ \text{T}.$$

$\boldsymbol{B}_p$ is in the positive x direction.

71. Since the radius is R = 0.0013 m, then the i = 50 A produces $B = \mu_0 i/2\pi R$ = 0.0077 T at the edge of the wire. The three equations, *Eqs.* 29-4, 29-17 and 29-20, agree at this point.

78. Using *Eq.* 29-20, $|\mathbf{B}| = (\mu_0 i/2\pi R^2)r$, we find that r = 0.00128 m gives the desired field value.

Ex.5-1: Pb.29-66.

重點整理－第 29 章 電流產生的磁場

必歐—沙伐定律 電流長度元 $i d\mathbf{s}$ 於離其距離 r 處產生的磁場 $d\vec{B}=\frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\hat{r}}{r^2}$, μ_0 (= $4\pi\times10^{-7}$ T·m/A, N/A^2, Wb/A·m, H/m)為真空之磁導率, $\mathbf{r}$ 為電流長度元至觀測點之位置向量。

Note 電流產生的磁場強度必正比於電流。

長直導線產生的磁場 $B=\frac{\mu_0 i}{2\pi r}$, r:觀測點至導線之垂直距離, $\mathbf{B}$ 方向 $\mathbf{J}\times\mathbf{r}$。磁場線為共軸同心圓。

直導線(長度 L)產生的磁場 (垂直平分線上)

$$B=\frac{\mu_0 i}{2\pi r}\frac{L/2}{\sqrt{r^2+(L/2)^2}},\ \mathbf{B}\text{ 方向 }\mathbf{J}\times\mathbf{r}。$$

圓弧導線產生的磁場 $B=\frac{\mu_0 i\phi}{4\pi R}$,

(圓弧中心, ϕ:圓弧張角, R:圓弧半徑)

兩平行電流(i_1 & i_2)之作用力 $F=\frac{\mu_0 i_1 i_2}{2\pi d}L$,

d: 兩電流之距, L: 導線長度; **力方向**: 電流同向則相吸, 反向則相斥。

$$\textbf{F1}.\int\frac{dx}{(a^2+x^2)^{3/2}}=\frac{x}{a^2(a^2+x^2)^{1/2}}\ \text{(Hint } x=a\tan\beta)$$

•**備忘錄**•

安培電流之定義 兩平行長直導線各載相同電流, 當其相距 1 m 時, 互施於對方每單位長度之力若為 2×10^{-7} N, 則此電流規定為 1 A (1946 年)。

安培定律 $\oint\vec{B}\cdot d\vec{\ell}=\mu_0 i_{enc}$, (沿著任意封閉路徑磁場之線積分值等於該路徑包圍之淨電流乘以μ_0).

理想螺線管之磁場 (導線沿著圓柱面緊密纏繞, 內部之磁場為均勻的, $\mathbf{B}$ 方向沿著對稱軸)

$B=\mu_0 n i$, n: 線圈密度(單位長度之匝數).

環形線圈之磁場 $B=\frac{\mu_0 iN}{2\pi}\frac{1}{r}$, N: 線圈匝數.

圓形導線之磁場 即圓形電流迴路, z 為中心軸

$$B_z=\frac{\mu_0 iR^2}{2(R^2+z^2)^{3/2}};\ z=0\text{ (中心)},\ B_z=\mu_0 i/2R;$$

As $z \gg a$, $B_z=\frac{\mu_0 iR^2}{2z^3}=\frac{\mu_0\mu}{2\pi z^3}$ (field due to a dipole).

如前所述，腦部活動怎樣產生磁場？

Law of Biot and Savart 必歐-沙伐定律; current-length element 電流長度元; permeability 磁導率; Ampére 安培; Ampere law 安培定律; Amperian loop 安培迴線; wire/ conductor 導線; circular wire 圓形導線; solenoid 螺線管; toroid 環(環形線圈); rail gun 導軌火炮; conducting fuse 熔線; MEG 腦照相術; SQUID 超導量子干射儀;