Chapter ***28*** ***Magnetic Fields***

Few subject in science are more difficult to understand than magnetism.

03. (**a**) The force on the electron is $\boldsymbol{F}_B = q\boldsymbol{v}\times\boldsymbol{B} = q(v_x\mathbf{i}+v_y\mathbf{j})\times(B_x\mathbf{i}+B_y\mathbf{j}) = q(v_xB_y-v_yB_x)\mathbf{k} = (-1.6\times10^{-19})[(2.0\times10^6)(-0.15) - (3.0\times10^6)(0.030)]\mathbf{k} = (6.2\times10^{-14}\text{ N})\mathbf{k}$. Thus, the magnitude of $\boldsymbol{F}_B$ is 6.2×10^{-14} N, and $\boldsymbol{F}_B$ points in the positive z direction. (**b**) This amounts to repeating the above computation with a change in the sign in the charge. Thus, $\boldsymbol{F}_B$ has the same magnitude but points in the $-z$ direction, namely, $\boldsymbol{F}_B = -(6.2\times10^{-14}\text{ N})\mathbf{k}$.

08. We apply $\boldsymbol{F} = q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) = m_e\boldsymbol{a}$ to solve for $\boldsymbol{E}$:

$$\vec{E} = \frac{1}{q}m_e\vec{a} - \vec{v}\times\vec{B} = \frac{1}{q}m_e\vec{a} + \vec{B}\times\vec{v}$$

$$= \frac{(9.11\times10^{-31}\text{ kg})(2.00\times10^{12}\text{ m/s}^2)}{-1.60\times10^{-19}\text{ C}}\hat{\text{i}}$$

$$+ (400\ \mu\text{T})\hat{\text{i}}\times[(12.0\text{ km/s})\hat{\text{j}}+(15.0\text{ km/s})\hat{\text{k}}]$$

$$= (-11.4\hat{\text{i}} - 6.00\hat{\text{j}} + 4.80\hat{\text{k}})\text{ V/m}.$$

12. We use Eq. 28-12 to solve for V:

$$V = \frac{1}{n\ell e}iB = 7.4\times10^{-6}\text{ (V)}$$

$$= \frac{(23)(0.65)}{(8.47\times10^{28})(150\times10^{-6})(1.6\times10^{-19})}.$$

15. From Eq. 28-16, we find

$$B = \frac{m_e v}{er} = \frac{(9.11\times10^{-31})(1.30\times10^6)}{(1.60\times10^{-19})(0.350)} = 2.11\times10^{-5}\text{ (T)}.$$

19. (**a**) The frequency of revolution is

$$f = \frac{1}{2\pi m_e}Bq = 9.78\times10^5\text{ Hz}$$

$$= \frac{(35.0\times10^{-6}\text{ T})(1.60\times10^{-19}\text{ C})}{2\pi(9.11\times10^{-31}\text{ Kg})}.$$

(**b**) Using Eq. 28-16, we obtain

$$r = \frac{1}{qB}m_e v = \frac{1}{qB}\sqrt{2m_eK} = 0.964\text{ m}$$

$$= \frac{\sqrt{2(9.11\times10^{-31}\text{ kg})(100\text{ eV})(1.60\times10^{-19}\text{ C})}}{(1.60\times10^{-19}\text{ C})(35.0\times10^{-6}\text{ T})}.$$

24. We consider the point at which it enters the field-filled region, velocity vector pointing downward. The field points out of the page so that $\boldsymbol{v}\times\boldsymbol{B}$ points leftward, which indeed seems to be the direction it is "pushed''; therefore, $q > 0$ (it is a proton). (**a**) Eq. 28-17 becomes $T = 2\pi m_p/eB$, or

$$2(130\times10^{-9}) = 2\pi(1.67\times10^{-27})/[(1.60\times10^{-19})B],$$

which yields $|\boldsymbol{B}| = 0.252$ T. (**b**) Doubling the kinetic energy implies multiplying the speed by $2^{1/2}$. Since the period T does not depend on speed, then it remains the same (even though the radius increases by a factor of $2^{1/2}$). Thus, $t = T/2 = 130$ ns, again.

33. (**a**) The magnitude of the magnetic force on the wire is given by $F_B = iLB\sin\phi$, where i is the current in the wire, L is the length of the wire, B is the magnitude of the magnetic field, and ϕ is the angle between the current and the field. In this case $\phi = 70°$. Thus,

$$F_B = (5000\text{ A})(100\text{ m})(60.0\times10^{-6}\text{ T})\sin70° = 28.2\text{ N}.$$

(**b**) We apply the right-hand rule to the vector product $\boldsymbol{F}_B = i\boldsymbol{L}\times\boldsymbol{B}$ to show that the force is to the west.

35. (**a**) The magnetic force on the wire must be upward and have a magnitude equal to the gravitational force mg on the wire. Since the field and the current are perpendicular to each other the magnitude of the magnetic force is given by $F_B = iLB$, where L is the length of the wire. Thus,

$$iLB = mg \Rightarrow i = mg/LB = (0.0130\text{ kg})(9.80\text{ m/s}^2)/[(0.620\text{ m})(0.440\text{ T})] = 0.467\text{ A}.$$

(**b**) Applying the right-hand rule reveals that the current must be from left to right.

40. We establish coordinates such that the two sides of the right triangle meet at the origin, and the $\ell_y = 50$ cm side runs along the $+y$ axis, while the $\ell_x = 120$ cm side runs along the $+x$ axis. The angle made by the hypotenuse (of length 130 cm) is $\theta = \tan^{-1}(50/120) = 22.6°$, relative to the 120 cm side. If one measures the angle counterclockwise from the $+x$ direction, then the angle for the hypotenuse is $180° - 22.6° = +157°$. Since we only seek to find the magnitudes of the forces, we have the freedom to assume the current is flowing, say, counterclockwise in the triangular loop (as viewed by an observer on the $+z$ axis). We take $\boldsymbol{B}$ to be in the same direction as that of the current flow in the hypotenuse. Then, with $B = |\boldsymbol{B}| = 0.0750$ T, $B_x = -B\cos\theta = -0.0692$ T and $B_y = B\sin\theta = 0.0288$ T. (**a**) Eq. 28-26 produces zero force when $\boldsymbol{L}\|\boldsymbol{B}$ so there is no force exerted on the hypotenuse of length 130 cm. (**b**) On the 50 cm side, the B_x component produces a force $i\ell_yB_x\mathbf{k}$ and there is no contribution from the B_y component. The magnitude of the force on the ℓ_y side is therefore

$$(4.00\text{ A})(0.500\text{ m})(0.0692\text{ T}) = 0.138\text{ N}.$$

(**c**) On the 120 cm side, the B_y component produces a force $i\ell_xB_y\mathbf{k}$ and there is no contribution from the B_x component. Using SI units, the magnitude of the force on the ℓ_x side is also

$$(4.00\text{ A})(1.20\text{ m})(0.0288\text{ T}) = 0.138\text{ N}.$$

(**d**) The net force is $i\ell_yB_x\mathbf{k} + i\ell_xB_y\mathbf{k} = 0$, keeping in mind that $B_x < 0$ due to our initial assumptions. If we had instead assumed $\boldsymbol{B}$ went the opposite direction of the current flow in the hypotenuse, then $B_x > 0$ but $B_y < 0$ and a zero net force would still be the result.

47. (a) The magnitude of the magnetic dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current in each turn, and A is the area of a loop. In this case the loops are circular, so $A = \pi r^2$, where r is the radius of a turn. Thus

$$i = \frac{\mu}{N\pi r^2} = \frac{2.30\ \mathrm{A\cdot m^2}}{(160)(\pi)(0.0190\ \mathrm{m})^2} = 12.7\ \mathrm{A}.$$

(b) The maximum torque occurs when the dipole moment is perpendicular to the field (or the plane of the loop is parallel to the field). It is given by

$\tau_{max} = \mu B = (2.30\ \mathrm{A{\cdot}m^2})(35.0\times10^{-3}\ \mathrm{T}) = 8.05\times10^{-2}\ \mathrm{N{\cdot}m}.$

49. (**a**) The area of the loop is A = (½)(30 cm)(40 cm) = $6.0\times10^2\ \mathrm{cm^2}$, so, so

$$\mu = iA = (50\ \mathrm{A})(6.0\times10^{-2}\ \mathrm{m^2}) = 0.30\ \mathrm{A{\cdot}m^2}.$$

(**b**) The torque on the loop is

$$\tau = \mu B\sin\theta = (0.30\ \mathrm{A{\cdot}m^2})(80\times10^{-3}\ \mathrm{T}) = 2.4\times10^{-2}\ \mathrm{N{\cdot}m}.$$

51. (**a**) The magnitude of the magnetic moment vector is

$$\mu = \Sigma_n i_n A_n = (\pi r_1^2)\, i_1 + (\pi r_2^2)\, i_2 = \pi(7.00\ \mathrm{A})\times[(0.200\ \mathrm{m})^2 + (0.300\ \mathrm{m})^2] = 2.86\ \mathrm{A{\cdot}m^2}.$$

(**b**) Now,

$$\mu = \Sigma_n i_n A_n = (\pi r_2^2)\, i_2 - (\pi r_1^2)\, i_1 = \pi(7.00\ \mathrm{A})\times[(0.300\ \mathrm{m})^2 - (0.200\ \mathrm{m})^2] = 1.10\ \mathrm{A{\cdot}m^2}.$$

87.* The total magnetic force on the loop L is

$$\vec{F}_B = i\oint_L (d\vec{L}\times\vec{B}) = i(\oint_L d\vec{L})\times\vec{B} = 0\,.$$

We note that $\oint_L d\vec{L} = 0$. If $\boldsymbol{B}$ is not a constant, however, then the equality $\oint_L (d\vec{L}\times\vec{B}) = (\oint_L d\vec{L})\times\vec{B}$ is not necessarily valid, so $\boldsymbol{F}_B$ is *not* always zero.

88.* (**a**) Since $\boldsymbol{B}$ is *uniform*,

$$\vec{F}_B = \int_{wire} i d\vec{L}\times\vec{B} = i(\int_{wire} d\vec{L})\times\vec{B} = i\vec{L}_{ab}\times\vec{B}\,,$$

where we note that $\int_{wire} d\vec{L} = \vec{L}_{ab}$, with $\vec{L}_{ab}$ being the displacement vector from a to b.

(**b**) Now $\boldsymbol{L}_{ab} = 0$, so $\boldsymbol{F}_B = i\boldsymbol{L}_{ab}\times\boldsymbol{B} = 0$.

magnetism/magnetics 磁學; magnetic field 磁場; tesla (T) 特士拉; magnetic force 磁力;crossed field 正交場; circulation 環行; Hall effect 霍耳效應; current loop 電流迴路; magnetic dipole 磁偶極; magnetic dipole moment 磁矩/磁偶極矩; electromagnet 電磁鐵; permanet magnet 永磁(永久磁鐵); commutator 整流器; gaussmeter 高斯計; cyclotron 迴旋加速器; sychrotron 同步加速器; mass spectrometer 質譜儀; galvanometer 電(檢)流計; pitch 螺距; magnetic bottle 磁瓶; auroral oval 極光帶; Van Allen radiation belts 范艾倫輻射帶; fast-neutron therapy 快中子療法; AMS 太空磁譜儀; CRT 陰極射線管; deuterium 氘; deuteron 氘/重氫核; audiotape 錄音帶; videotape 錄影帶; hard disk drive (HDD)磁碟機;

*最強磁石 NdFeB, 磁能積達 540 kJ/m^3.

*鐵鈷合金($Fe_{65}Co_{35}$)飽和磁通量密度(B_s)可達 2.45 T.

重點整理－第 28 章 磁場

磁場 由帶電質點(電荷 q,速度 $\boldsymbol{v}$)受力定義

$\boldsymbol{F}_B = q\boldsymbol{v}\times\boldsymbol{B}$ or $F_B = qvB\sin\theta$，單位(SI)：$\mathrm{N{\cdot}C^{-1}{\cdot}m^{-1}{\cdot}s} = \mathrm{N{\cdot}A^{-1}{\cdot}m^{-1}} \equiv$ tesla (T), Note ***a***.磁力 $\boldsymbol{F}_B\perp\boldsymbol{v}$ 及 $\boldsymbol{F}_B\perp\boldsymbol{B}$，***b***.$(d/dt)(\boldsymbol{v}\cdot\boldsymbol{v}) = 2\boldsymbol{v}\cdot d\boldsymbol{v}/dt = (2/m)\boldsymbol{v}\cdot\boldsymbol{F}_B = 0, |\boldsymbol{v}| = \text{const.}$ 即等速率運動(**磁力只能使帶電質點偏向**)。1 T (SI) = 10^4 G (cgs), 1 mG = 10^{-3} G, 基隆地磁 0.35824 G.

Note 磁場線必呈封閉曲線

正交場 電場垂直磁場稱之，

帶電質點受力 = 電力 + 磁力 $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$,

當合力為零時, $\boldsymbol{E} = -\boldsymbol{v}\times\boldsymbol{B}$ 或 $E = vB$。

霍爾效應 當放置於磁場 B 中板狀導電體通電流 i 時, 電荷載子受磁力而偏向並累積於側邊, 引起兩側邊電位差(霍爾電壓) $V_H = E_H w = v_d B w = iB/nqt$, w 及 t 為平行磁場面之導電體寬度及厚度。依此測霍爾電壓可得磁場大小(高斯計)。

於磁場中環行(磁力提供向心力) ***a***.當 $\boldsymbol{v}\perp\boldsymbol{B}$, 帶電質點作等速率圓周運動($r$:半徑, 環行方向遵循反磁性), $\boldsymbol{F}_B = |q|vB = m(v^2/r) = \boldsymbol{F}_c$, $|q|Br = mv$ = 動量之大小, $r = mv/|q|B$, $\omega = v/r = |q|B/m$, $T = 2\pi r/v = 2\pi m/|q|B$; ***b***.當 $\boldsymbol{v}\cdot\boldsymbol{B}\neq 0$, $\boldsymbol{v}_{\parallel}$ (= const, $\parallel\boldsymbol{B}$) + $\boldsymbol{v}_{\perp}$ ($\perp\boldsymbol{B}$, 等速率圓周運動) ⇒ 螺旋軌跡, 螺距 $p = v_{\parallel}T$。

載電流導線所受之磁力 各小段導線 $d\boldsymbol{L}$ 所受之力 $d\boldsymbol{F} = id\boldsymbol{L}\times\boldsymbol{B}$, ***a***.直導線及均勻磁場時 $\boldsymbol{F}_B = i\boldsymbol{L}\times\boldsymbol{B}$, ***b***.均勻磁場時, $\boldsymbol{F} = \int d\boldsymbol{F} = i(\int d\boldsymbol{L})\times\boldsymbol{B} = i\Delta\boldsymbol{L}\times\boldsymbol{B}$, 對封閉導線, 所受磁力為零 $\boldsymbol{F} = 0$ ($\because \int_O d\boldsymbol{L} = 0$)。

電流迴路之等效磁(偶極)矩 $\boldsymbol{\mu} \equiv iA\boldsymbol{n}$, i:電流, $A\boldsymbol{n}$: 面積向量(方向由右手定則決定),單位: $\mathrm{A{\cdot}m^2}$ or J/T. 電流迴路等同磁偶極(小磁針或柱狀磁鐵,§29.6)。多匝迴路 $\mu = NiA$, N: 線圈匝數,

電流迴路所受之力矩 $\boldsymbol{\tau} = \boldsymbol{\mu}\times\boldsymbol{B}$ or $\tau = \mu B\sin\theta$。

磁偶極之指向能 $U = -\boldsymbol{\mu}\cdot\boldsymbol{B} = -\mu B\cos\theta$。

怎樣才能產生高能量的中子束？

*說文解字 **MAGNET** (磁石,磁鐵): 此字由來有兩種說法，(***a***)因人—相傳某年前的某一天，有一位叫 Magnes 之牧羊人趕羊時，鐵杖被某特殊"石頭"吸住，從此人類進入磁的世紀，(***b***)因地—中亞小亞細亞有個地方 Magnesia，盛產磁鐵礦(magnetite, Fe_3O_4)。

*1950s, 日常生活使用二個磁鐵, 一為腳踏車另一為雨刷馬達; 現今, 家用 150 個及車用 70 個磁鐵。

Ex.4-1: Pb.28-42 *&* ***Ex.4-2: Pb.28-74****.*

S1. 瑞法邊界之大強子對撞機(Large Hadron Collider)可將一顆質子加速至 7000 GeV, 兩顆互撞就有 14000 GeV (美); 費米實驗室可產生 1960 GeV 能量.

Richard Feynman claimed that *"ten thousand years from now, there can be little doubt that the most significant event of the 19th century will be judged as Maxwell's discovery of the laws of electro-dynamics"*.•**備忘錄**•

♦"電磁學的故事", 涂世雄/王雄正/蔡曜州, 科學發展 378 期 (2004 年 6 月) 62。♦"**交流電之父—特士拉**(Tesla)",卡爾森,科學人 No. 38 (2005 年 4 月號),pp.98-106。♦"**天才的悲哀與喜悅－安培與電動力學**",張文亮,科學發展 364 期 (9204) 56。♦"**地球是個發電機**—真實與虛擬共舞",趙丰,科學人 No.39 (2005 年 5 月) pp.32-34。♦ "**地球磁場即將反轉?**"格拉茲麥爾、歐爾森,科學人 No.39 (2005 年 5 月) pp.36-43。♦*Q&A.* **磁鐵斷開來後為什麼翻面才會相吸**?科學人 No. 41 (2005 年 7 月號) 123。♦認識磁力, DSC. ♦電磁風暴/大強子對撞機, DSC & NGC.