Chapter 27 Circuits

02. The chemical energy of the battery is reduced by $\Delta E = q\varepsilon$, where q is the charge that passes through in time $\Delta t = 6.0$ min, and ε is the emf of the battery. If i is the current, then $q = i\,\Delta t$ and

$$E = I\,\varepsilon\,\Delta t = (5.0\text{ A})(6.0\text{ V})(6.0\text{ min})(60\text{ s/min}) = 1.1\times 10^4\text{ J}.$$

03. If P is the rate at which the battery delivers energy and Δt is the time, then $\Delta E = P\Delta t$ is the energy delivered in time Δt. If q is the charge that passes through the battery in time Δt and ε is the emf of the battery, then $\Delta E = q\varepsilon$. Equating the two expressions for ΔE and solving for Δt, we obtain

$$\Delta t = \frac{q\varepsilon}{P} = \frac{(120\text{Ah})(12.0\text{V})}{100\text{W}} = 14.4\text{ h}.$$

06. The current in the circuit is

$$i = (150\text{ V} - 50\text{ V})/(3.0\ \Omega + 2.0\ \Omega) = 20\text{ A}.$$

So from $V_Q + 150\text{ V} - (2.0\ \Omega)i = V_P$, we get $V_Q = 100\text{ V} + (2.0\ \Omega)(20\text{ A}) - 150\text{ V} = -10$ V.

18. (***a***) $R_{eq}(FH) = (10.0)(10.0)(5.00)/[(10.0)(10.0) + 2(10.0)(5.00)] = 2.50\ (\Omega)$. (***b***) $R_{eq}(FG) = (5.00)R/(R+5.00)$, where $R = 5.00 + (5.00)(10.0)/(5.00+10.0) = 8.33\ (\Omega)$. So $R_{eq}(FG) = (5.00)(8.33)/(5.00+8.33) = 3.13\ (\Omega)$.

23. First, we note V_4, that the voltage across R_4 is equal to the sum of the voltages across R_5 and R_6:

$$V_4 = i_6(R_5+R_6) = (1.40)(8.00+4.00) = 16.8\text{ (V)}.$$

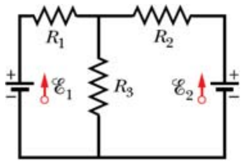
The current through R_4 is then equal to $i_4 = V_4/R_4 = 16.8/16.0 = 1.05$ (A). By the junction rule, the current in R_2 is $i_2 = i_4 + i_6 = 1.05 + 1.40 = 2.45$ (A), so its voltage is $V_2 = (2.00)(2.45) = 4.90$ (V). The loop rule tells us the voltage across R_3 is $V_3 = V_2 + V_4 = 21.7$ V (implying that the current through it is $i_3 = V_3/2.00 = 10.85$ A). The junction rule now gives the current in R_1 as $i_1 = i_2 + i_3 = 2.45+10.85 = 13.3$ (A), implying that the voltage across it is $V_1 = (13.3)(2.00) = 26.6$ (V). Therefore, by the loop rule, $\varepsilon = V_1 + V_3 = 26.6 + 21.7 = 48.3$ (V).

27. (***a***) We note that the R_1 resistors occur in series pairs, contributing net resistance $2R_1$ in each branch where they appear. Since $\varepsilon_2 = \varepsilon_3$ and $R_2 = 2R_1$, from symmetry we know that the currents through ε_2 and ε_3 are the same: $i_2 = i_3 = i$. Therefore, the current through ε_1 is $i_1 = 2i$. Then from $V_b - V_a = \varepsilon_2 - iR_2 = \varepsilon_1 + (2R_1)(2i)$ we obtain

$$i = \frac{\varepsilon_2 - \varepsilon_1}{4R_1 + R_2} = \frac{4.0-2.0}{4(1.0)+2.0} = 0.33\text{ (A)}.$$

Therefore, the current through ε_1 is $i_1 = 2i = 0.67$ A. (***b***) The direction of i_1 is downward. (***c***) The current through ε_2 is $i_2 = 0.33$ A. (***d***) The direction of i_2 is upward. (***e***) From part (a), we have $i_3 = i_2 = 0.33$ A. (***f***) The direction of i_3 is also upward. (***g***) $V_a - V_b = -iR_2 + \varepsilon_2 = -(0.333)(2.0) + 4.0 = 3.3$ (V).

33. (a) We first find the currents. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is to the left. Let i_3 be the current in R_3 and take it to be positive if it is upward. The junction rule produces



$$i_1 + i_2 + i_3 = 0.$$

The loop rule applied to the left-hand loop produces

$$\varepsilon_1 - i_1 R_1 + i_3 R_3 = 0,$$

and applied to the right-hand loop produces

$$\varepsilon_2 - i_2 R_2 + i_3 R_3 = 0.$$

We substitute $i_3 = -i_2 - i_1$, from the first eq., into the other two to obtain

$$\varepsilon_1 - i_1 R_1 - i_2 R_2 - i_3 R_3 = 0$$

and

$$\varepsilon_2 - i_2 R_2 - i_2 R_3 - i_1 R_3 = 0.$$

Solving the above equations yield

$$i_1 = \frac{\varepsilon_1(R_2+R_3) - \varepsilon_2 R_3}{R_1R_2 + R_1R_3 + R_2R_3} = 0.421\text{ (A)}$$

$$= \frac{(3.00)(2.00+5.00)-(1.00)(5.00)}{(4.00)(2.00)+(4.00)(5.00)+(2.00)(5.00)}.$$

$$i_2 = \frac{\varepsilon_2(R_1+R_3) - \varepsilon_1 R_3}{R_1R_2 + R_1R_3 + R_2R_3} = -0.158\text{ (A)}$$

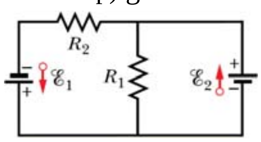
$$= \frac{(1.00)(4.00+5.00)-(3.00)(5.00)}{(4.00)(2.00)+(4.00)(5.00)+(2.00)(5.00)}.$$

$$i_3 = -\frac{\varepsilon_2 R_1 + \varepsilon_1 R_2}{R_1R_2 + R_1R_3 + R_2R_3} = -0.263\text{ (A)}$$

$$= -\frac{(1.00)(4.00)+(3.00)(2.00)}{(4.00)(2.00)+(4.00)(5.00)+(2.00)(5.00)}.$$

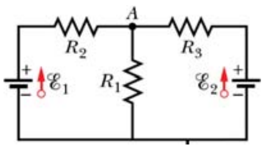
Note that the current i_3 in R_3 is actually downward and the current i_2 in R_2 is to the right. The current i_1 in R_1 is to the right. (***a***) The power dissipated in R_1 is $P_1 = i_1^2R_1 = (0.421)^2(4.00) = 0.709$ (W). (***b***) The power dissipated in R_2 is $P_2 = i_2^2R_2 = (-0.158)^2(2.00) = 0.0499 \approx 0.050$ (W). (***c***) The power dissipa-pated in R_3 is $P_3 = i_3^2R_3 = (-0.263)^2(5.00) = 0.346$ (W). (***d***) The power supplied by ε_1 is $i_3\varepsilon_1 = (0.421)(3.00) = 1.26$ (W). (***e***) The power "supplied" by ε_2 is $i_2\varepsilon_2 = (-0.158)(1.00) = -0.158$ (W). The negative sign indicates that ε_2 is actually absorbing energy from the circuit.

66. (**a**) The loop rule (proceeding counterclockwise around the right loop) leads to $\varepsilon_2 - i_1R_1 = 0$ (where i_1 was assumed downward). This yields $i_1 = 0.0600$ A. (**b**) The direction of i_1 is downward. (**c**) The loop rule (counterclockwise around the left loop) gives



$(+\varepsilon_1) + (+i_1R_1) + (-i_2R_2) = 0$, where i_2 has been assumed leftward. This yields $i_3 = 0.180$ A.(**d**) A positive value of i_3 implies that our assumption on the direction is correct, i.e., it flows leftward. (**e**) The junction rule tells us that the current through the 12 V battery is 0.180 + 0.0600 = 0.240 (A). (**f**) The direction is upward.

95. (**a**) Using the junction rule ($i_1 = i_2 + i_3$) we write two loop rule equations:



$\varepsilon_1 - i_2 R_2 - (i_2 + i_3)R_1 = 0$,

$\varepsilon_2 - i_3 R_3 - (i_2 + i_3)R_1 = 0$.

Solving, we find i_2 = 0.0109 A (rightward, as was assumed in writing the equations as we did), i_3 = 0.0273 A (leftward), and $i_1 = i_2 + i_3$ = 0.0382 A (downward). (**b**) downward. See the results in part (a). (**c**) i_2 = 0.0109 A. See the results in part (a). (**d**) rightward. See the results in part (a). (**e**) i_3 = 0.0273 A. See the results in part (a). (**f**) leftward. See the results in part (a). (**g**) The voltage across R_1 equals V_A: (0.0382 A)(100 Ω) = +3.82 V.

45. During charging, the charge on the positive plate of the capacitor is given by $q = C\varepsilon(1 - e^{-t/\tau})$, where C is the capacitance, ε is applied emf, and $\tau = RC$ is the capacitive time constant. The equilibrium charge is $q_{eq} = C\varepsilon$. We require $q = 0.99q_{eq} = 0.99C\varepsilon$, so $0.99 = 1-e^{-t/\tau}$. Thus, $e^{-t/\tau} = 0.01$. Taking the natural logarithm of both sides, we obtain $t/\tau = -\ln 0.01 = 4.61$ or $t = 4.61\tau$.

46. (***a***) $\tau = RC = (1.40\times10^{6})(1.80\times10^{-6}) = 2.52$ (s). (***b***) $q_o = \varepsilon C = (12.0)(1.80\mu) = 21.6\mu$C. (***c***) The time t satisfies $q = q_0(1 - e^{-t/RC})$, or

$$t = RC\ln\frac{q_0}{q_0 - q} = (2.52)\ln\frac{21.6}{21.6-16.0} = 3.40 \text{ (s)}.$$

48. Here we denote the battery emf as V. Then the requirement stated in the *Pb* that the resistor voltage be equal to the capacitor voltage becomes $iR = V_C$, or $Ve^{-t/RC} = V(1-e^{-t/RC})$, where Eqs. 27-34 and 35 have been used. This leads to $t = RC \ln 2$, or t = 0.208 ms.

53. At t = 0 the capacitor is completely uncharged and the current in the capacitor branch is as it would be if the capacitor were replaced by a wire. Let i_1 be the current in R_1 and take it to be positive if it is to the right. Let i_2 be the current in R_2 and take it to be positive if it is downward. Let i_3 be the current in R_3 and take it to be positive if it is downward. The junction rule produces $i_1 = i_2 + i_3$ the loop rule applied to the left-hand loop produces $\varepsilon_1 - i_1R_1 + i_2R_2 = 0$, and the loop rule applied to the right-hand loop produces $i_2R_2 - I_3R_3 = 0$.

Since the resistances are all the same we can simplify the mathematics by replacing R_1, R_2, and R_3 with R. (***a***) Solving the three simultaneous eqs., we find $i_1 = 2\varepsilon/3R = 2(1.2\times10^{3})/[3(0.73\times10^{6}) = 1.1\times10^{-3}$ A. (***b***) $i_2 = \varepsilon/3R = (1.2\times10^{3})/[3(0.73\times10^{6})] = 5.5\times10^{-4}$ A. (***c***) $i_3 = i_2 = 5.5\times10^{-4}$ A. At $t = \infty$ the capacitor is fully charged and the current in the capacitor branch is 0. Thus, $i_1 = i_2$, and the loop rule yields $\varepsilon_1 - i_1R_1 + i_1R_2 = 0$. (***d***) The solution is $i_1 = \varepsilon/2R = (1.2\times10^{3})/[2(0.73\times10^{6})] = 8.2\times10^{-4}$ A. (***e***) $i_2 = i_1 = 8.2\times10^{-4}$ A. (***f***) As stated before, the current in the capacitor branch is $i_3 = 0$. We take the upper plate of the capacitor to be positive. This is consistent with current flowing into that plate. The junction equation is $i_1 = i_2 + i_3$, and the loop equations are

$$\varepsilon - i_1R - i_2R = 0 \quad \text{and} \quad -(q/C) - i_3R + i_2R = 0.$$

We use the first eq. to substitute for i_1 in the second and obtain $\varepsilon - 2i_2R - i_3R = 0$. Thus $i_2 = (\varepsilon - i_3R)/2R$. We substitute this expression into the third eq. above to obtain $-(q/C) - (i_3R) + (\varepsilon/2) - (i_3R/2) = 0$. Now we replace i_3 with dq/dt to obtain

$$\frac{3R}{2}\frac{dq}{dt} + \frac{q}{C} = \frac{\varepsilon}{2}.$$

This is just like the eq. for an *RC* series circuit, except that the time constant is $\tau = 3RC/2$ and the impressed potential difference is $\varepsilon/2$. The solution is

$$q = \frac{1}{2}C\varepsilon(1-e^{-2t/3RC}).$$

The current in the capacitor branch is

$$i_3(t) = \frac{dq}{dt} = \frac{\varepsilon}{3R}e^{-2t/3RC}.$$

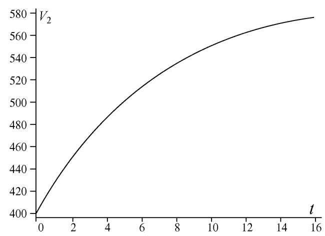
The current in the center branch is

$$i_2(t) = \frac{\varepsilon}{2R} - \frac{1}{2}i_3 = \frac{\varepsilon}{2R} - \frac{\varepsilon}{6R}e^{-2t/3RC} = \frac{\varepsilon}{6R}(3-e^{-2t/3RC}).$$

and the potential difference across R_2 is

$$V_2(t) = i_2R = \frac{1}{6}\varepsilon(3-e^{-2t/3RC}).$$

(***g***) For t = 0, $e^{-2t/3RC}$ is 1 and $V_2 = \varepsilon/3 = 1.2\times10^{3}/3 = 4.0\times10^{2}$ (V). (***h***) For $t = \infty$, $e^{-2t/3RC} = 0$ and $V_2 = \varepsilon/2 = (1.2\times10^{3}\text{ V})/2 = 6.0\times10^{2}$ V. (***i***) A plot of V_2 as a function of time is shown in the following graph.



54. In the steady state situation, the capacitor voltage will equal the voltage across R_2 = 15 kΩ:

$$V_0 = R_2\frac{\varepsilon}{R_1 + R_2} = (15.0)\frac{20.0}{10.0+15.0} = 12.0 \text{ (V)}.$$

Now, multiplying Eq. 27-39 by the capacitance leads to $V = V_0e^{-t/RC}$ describing the voltage across the capacitor (and across R_2 = 15.0 kΩ) after the switch is opened (at t = 0). Thus, with t = 0.00400 s, we obtain

$$V = (12)\exp[-0.004/(15.0\text{k}\times0.400\mu)] = 6.16 \text{ (V)}.$$

Therefore, using Ohm's law, the current through R_2 is $6.16/15000 = 4.11 \times 10^{-4}$ (A).

(如發現錯誤煩請告知, jyang@mail.ntou.edu.tw, Thanks.)

重點整理－第 27 章 直流電路

電動勢源(電源)—可對電荷載子作功並維持一定端電位差之元件；**電動勢**(emf, 單位: V) $\varepsilon = dW/dq$ 對單位正電荷由低電位端移至高位端所作的功，電路符號。**理想電動勢源**其內部無耗能機制，可提供端電位差ε；而**真實電動勢源**可視為一理想電動勢源ε"串聯"一內電阻 r，其提供端電位差$\varepsilon - iR$。**迴路**: 一封閉的導電路徑(自由電荷可作循環運動)。**接地**:透過導體與地面相接，電路符號，此為零電位。電阻器電路符號。

Kirchhoff's Junction Rule (Current Law)

流入或流出任一節點的電流之代數和須為零；流入任一節點的電流和須等於流出該節點的電流。<電荷守恆 charge conservation>

Kirchhoff's Loop Rule (Voltage Law)

繞著一電路之任一迴路走一圈，所遇到的電位改變量之代數和須為零。<能量守恆>

解電路問題技巧：**1**.劃出電路；**2**.對電路之各分路，給予一電流(符號及方向)；**3**.運用串聯及並聯方法，減少分路；**4**.利用節點規則以減少電流變數；**5**.利用迴路規則解出各電流變數。

S1.鎳氫或鋰類可充電電池以 mAh (毫安培小時)表示儲存電荷量, 1 mAh = 3.6 C。***S2*.用電安全**: 最重要守則—勿使身體接觸高低電位而使電流通過身體或過電流使用；用電不當，造成觸電，可能致命，或電路走火引起生命財產的損失！電擊之危險: (1) 引起心臟或肺部發生問題，(2)致命的燒傷。**家居電線**: 傳統的為二線改用三線(含接地線)最安全：其為火(平)/中性(平)/接地(圓)線，或者使用較安全二線"極性插頭"[寬(中性)口及窄(火)口]。

電位差(電壓)符號規則：一般規則—電位減少(增加)，$-(+)V$；***a***.順(逆)著電流通過**電阻器**，$-(+)\ iR$；***b***.從**電容器**正(負)板移動到負(正)板，$-(+)q/C$；***c***.從**電源**正(負)端移動到負(正)端，$-(+)\varepsilon$。

電阻器串聯：***a***.流經各電阻器之電流皆相等，***b***.總電位差等於各電阻器之電位差和 $V_{ab} = \Sigma_i V_i$，***c***.等效電阻 $R_{eq} = \Sigma_i R_i$，$R_{eq} > R_i$。

電阻器並聯：***a***.各電阻器兩端之電位差皆相等，***b***.總電流等於經各電阻器之電流和 $I = \Sigma_i I_i$，***c***.等效電阻 $R_{eq}^{-1} = \Sigma_i R_i^{-1}$，$R_{eq} < R_i$。

RC* 電路**—電阻器 R 串聯電容器 C (簡易的)：a***.充電($q_0 = 0$) $q = C\varepsilon(1-e^{-t/\tau})$；***b***.放電($q_0 = C\varepsilon$) $q = C\varepsilon e^{-t/\tau}$；***c*.時間常數** $\tau \equiv RC$ ；***d***.電容器充電時，電阻器及電容器各分享一半電源提供之能量。

Note 電容器如未帶電(如剛充電時)，$V_C = 0$，即**短路**，等效電阻 $R_C = 0$；若飽和充電，$I_C = 0$，即**斷路**，等效電阻 $R_C \to \infty$。電容器充電或放電時其帶電量及電壓與時間關係為指數的。

伏特計(等效電阻 $R_V \to \infty$)測電壓用, 聯接須**並聯**; **安培計**(等效電阻 $R_A \to 0$)測電流用, 聯接須**串聯**。

須採取何種預防措施才能防止此類火災？

direct-current (DC) circuits 直流電路; device 裝置,器件; electromotive force (emf)電動勢; rechargeable 可充電的; loop 迴路(線); junction/node 節點; ground 接地; hot/neu- tral line 火/中性線; voltage 電壓,伏特數; (capaci- tive) time constant (電容)時間常數; ammeter 安培計; voltmeter 伏特計; polarized plug 極性插頭; electrical engineering 電機工程; electric grid 電力網; electric generator 發電機; pit stop 修理站; electric eel 電鰻; *Q&A*.電鰻如何發電？為什麼不會電到自己？科學人 2006 年 4 月號。*鋰電池：安全不起火，科學人 2007 年 1 月。•**備忘錄**•