Chapter **25** ***Capacitance***

01. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus, $q = (25\times10^{-6}\text{ F})(120\text{ V}) = 3.0\times10^{-3}\text{ C}$.

04. We use $C = A\varepsilon_0/d$. (**a**) Thus, $d = \varepsilon_0 A/C = (8.85\times10^{-12})(1.00)/(1.00) = 8.85\times10^{-12}$ (m). (**b**) Since d is much less than the size of an atom ($\sim10^{-10}$ m), this capacitor cannot be constructed.

05. Assuming conservation of volume, we find the radius R' of the combined spheres, then use $C' = 4\pi\varepsilon_0 R'$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V' = 2(4\pi/3)R^3$. The new radius R' is given by

$$(4\pi/3)R'^3 = 2(4\pi/3)R^3 \Rightarrow R' = 2^{1/3}R.$$

The new capacitance is $C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3}R = 5.04\pi\varepsilon_0 R$. With $R = 2.00$ mm, we obtain $C' = 5.04\pi(8.85\times10^{-12}\text{F/m})(2.00\times10^{-3}\text{m}) = 2.80\times10^{-13}\text{F}$.

12. (**a**) The potential difference across C_1 is $V_1 = 10.0$ V. Thus, $q_1 = C_1V_1 = (10.0\mu\text{F})(10.0\text{V}) = 1.00\times10^{-4}$ C. (**b**) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{eq} = C + \frac{C_2C}{C+C_2} = 1.50C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C+C_{eq}} = \frac{CV_1}{C+1.50C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(10.0\text{ V}/5) = 2.00\times10^{-5}\text{ C}.$$

14. The two 6.0 μF capacitors are in parallel and are consequently equivalent to $C_{eq} = 12\ \mu$F. Thus, the total charge stored (before the squeezing) is $q_{tot} = C_{eq}V_B = 120\mu$C. (**a**) and (**b**) As a result of the squeezing, one of the capacitors is now 12 μF (due to the inverse proportionality between C and d in Eq. 25-9) which represents an increase of 6.0 μF and thus a charge increase of

$$\Delta q_{tot} = \Delta C_{eq}V_B = (6.0\ \mu\text{F})(10\text{ V}) = 60\ \mu\text{C}.$$

24. Using Eq. 25-25 and $V = 1.00\text{ m}^3$, the energy stored is $U = uV = (½)\varepsilon_0E^2V$

$$= \tfrac{1}{2}(8.85\times10^{-12})(150)^2(1.00) = 9.96\times10^{-8}\text{ (J)}.$$

32. We use $E_R = q/4\pi\varepsilon_0R^2 = V_R/R$. Thus

$$u = \tfrac{1}{2}\varepsilon_0E^2 = \tfrac{1}{2}\varepsilon_0(V_R/R)^2$$

$$= (½)(8.85\times10^{-12})(8000/0.050)^2 = 0.11\text{ (pJ/m}^3).$$

34. If the original capacitance is given by $C = \varepsilon_0A/d$, then the new capacitance is $C' = \kappa\varepsilon_0A/2d$. Thus $C'/C = \kappa/2$ or $\kappa = 2C'/C = 2(2.6\text{ pF}/1.3\text{ pF}) = 4.0$.

42. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units), $C = C_1 + C_2 = 8.41\times10^{-12}$ (F)

$$= \frac{\varepsilon_0(A/2)\kappa_1}{d} + \frac{\varepsilon_0(A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\frac{\kappa_1+\kappa_2}{2}$$

$$= \frac{(8.85\times10^{-12})(5.56\times10^{-4})}{5.56\times10^{-3}}\frac{7.00+12.00}{2}.$$

43. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower/uppser half of the regions between the plates are $E_{1,2} = q/\kappa_{1,2}\varepsilon_0A$, where A is the plate area. Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1d}{2} + \frac{E_2d}{2} = \frac{qd}{2\varepsilon_0A}\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right) = \frac{qd}{2\varepsilon_0A}\frac{\kappa_1+\kappa_2}{\kappa_1\kappa_2},$$

So $$C = \frac{q}{V} = \frac{2\varepsilon_0A}{d}\frac{\kappa_1\kappa_2}{\kappa_1+\kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1\varepsilon_0A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1. With $A = 7.89\times10^{-4}\text{ m}^2$, $d = 4.62\times10^{-3}$ m, $\kappa_1 = 11.0$, and $\kappa_2 = 12.0$, the capacitance is, (in SI units) $= 1.73\times10^{-11}$ (F).

$$C = \frac{2(8.85\times10^{-12})(7.89\times10^{-4})}{4.62\times10^{-3}}\frac{(11.0)(12.0)}{11.0+12.0}.$$

45. (**a**) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\varepsilon_0A/C$ and $E = VC/\kappa\varepsilon_0A$

$$E = \frac{(50)(100\times10^{-12})}{5.4(8.85\times10^{-12})(100\times10^{-4})} = 1.0\times10^4\text{ (V/m)}.$$

(**b**) The free charge on the plates is $q_f = CV = (100\times10^{-12})(50) = 5.0\times10^{-9}$ (C). (**c**) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0A} + \frac{q_f}{2\varepsilon_0A} - \frac{q_i}{2\varepsilon_0A} - \frac{q_i}{2\varepsilon_0A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the last is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$q_i = q_f - \varepsilon_0 AE = 5.0\times10^{-9} - (8.85\times10^{-12})(100\times10^{-4})(100\times10^{4}) = 4.1\times10^{-9}\ \text{(C)} = 4.1\ \text{(nC)}.$$

46. (**a**) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d-b+\frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = 1.34\times10^{-11}\ \text{(F)} = 13.4\ \text{(pF)}$$

$$= \frac{(8.85\times10^{-12})(115\times10^{-4})(2.61)}{(2.61)(0.0124-0.00780)+0.0780}.$$

(**b**) $q = CV = (13.4\times10^{-12}\ \text{F})(85.5\ \text{V}) = 1.15$ nC. (**c**) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}}{(8.85\times10^{-12})(115\times10^{-4})} = 1.13\times10^{4}\ \text{(N/C)}.$$

(**d**) Using Eq. 25-34, we obtain

$$E_2 = E_1/\kappa = 1.13\times10^{4}/2.61 = 4.33\times10^{3}\ \text{(N/C)}.$$

70.• The voltage across capacitor 1 is

$$V_1 = q_1/C_1 = 30\ \mu\text{C}/10\ \mu\text{F} = 3.0\ \text{V}.$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2V2 = (20\ \mu\text{C})(3.0\ \text{V}) = 60\ \mu\text{C}.$$

which means a total of 90 μC of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of 90 μC of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45\ \mu$C. Thus, the voltage across capacitor 3 is

$$V_3 = q_3/C_3 = 45\ \mu\text{C} / 20\ \mu\text{F} = 2.3\ \text{V}.$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

78.• (**a**) The voltage across C_1 is 12 V, so the charge is $q_1 = C_1V_1 = 24\ \mu$C. (**b**) We reduce the circuit, starting with C_4 and C_3 (in parallel) which are equivalent to 4μF. This is then in series with C_2, resulting in an equivalence equal to (4/3)μF, which would have 12 V across it. The charge on this (4/3)μF capacitor (and therefore on C_2) is (4/3)μF(12 V) = 16 μC. Consequently, the voltage across C_2 is $V_2 = q_2/C_2 = 16\ \mu\text{C}/2\ \mu\text{F} = 8$ V. This leaves 12 V – 8 V = 4 V across C_4 (similarly for C_3).

Note 電容 *C* 勿與電荷單位 C 混淆！

Note 真空：$\kappa = 1$、金屬：$\kappa \to \infty$

重點整理－第 25 章 電 容

電容器：兩導體由絕緣體隔開之結構；用以儲存電荷及電能；電路符號–||–；**電板**：可容納電荷之導體，帶電量: q；因**導體內部及表面之電位相等**，兩電板存在**電位差** $V_a - V_b \equiv V_{ab}$ (or ΔV)，$q \propto E \propto V_{ab} \Rightarrow q \propto V_{ab}$；**電容** $C \equiv q/V_{ab}$，**單位** F, 1 F ≡ 1 C/V, 1 μF = 10^{-6} F, 1 pF = 10^{-12} F。

如何計算電容：***a***.假設電容器帶電量 q；***b***.計算兩電板間電場 $\boldsymbol{E}$ (可用高斯定律)；***c***.由電場計算兩電板間之電位差 V_{ab}；***d***.由 q / V_{ab} 得電容 C。

平行板電容器 $C = \varepsilon_0 A / d$; 兩電板間電場為均勻的, $E = \sigma / \varepsilon_0 = q / A\varepsilon_0$, 兩電板間<u>電位差</u>

$$V_{ab} = Ed = qd / A\varepsilon_0，C = q / V_{ab} = \varepsilon_0 A / d。$$

圓柱型電容器(中心軸長 L，內,外半徑 a,b)

$$C = 2\pi\varepsilon_0 L / \ln(b/a)。$$

球型電容器(內,外半徑 a,b) $C = 4\pi\varepsilon_0 ab/(b-a)$。

Note 圓柱及球型電容器當板距極小時，$C = \varepsilon_0 A/d$。

電容器並聯：***a***.各電容器兩端之電位差皆相等 $V_i = V$；***b***. $q_{tot} = \Sigma_i q_i$；***c***. $C_{eq} = \Sigma_i C_i$，$C_{eq} > C_i$。

電容器串聯：***a***.各電容器之帶電量皆相同 $q_i = q$；***b***. $V = \Sigma_i V_i$；***c***. $C_{eq}^{-1} = \Sigma_i C_i^{-1}$，$C_{eq} < C_i$。

Note 公式: 電容器**串**(並)聯如同電阻器**並**(串)聯。

電容器儲存的能量

$$U_e = \frac{1}{2}\frac{1}{C}q^2 = \frac{1}{2}CV^2 = \frac{1}{2}qV\ (真空)。$$

電場儲存的能量—電能密度 $u_e = (½)\varepsilon_0 E^2$ (真空)。

介電質：①.極性分子: 具永久性電偶極；②.非極性分子: 無永久性電偶極。在外施電場作用下，非極性介電質可"**極化**"，即生感應電偶極；兩類介電質在外施電場 $\boldsymbol{E}_0$ 作用下，皆可產生感應電場 $\boldsymbol{E}_{ind}$ 以減弱外施電場，此效應可用介電常數以描述；**介電常數** κ (> 1)定義為"E_0(真空內之電場)" / "E_D(介電質之電場)"，即在介電質內，(淨)電場變為真空的 κ^{-1}，介電質內：$\boldsymbol{E}_D$ (淨) = $\boldsymbol{E}_0 + \boldsymbol{E}_{ind}$ or $E_D = E_0 - E_{ind} = E_0 / \kappa < E_0$ ($E_D, E_{ind}, E_0 > 0$)。

電容器中塞入介電質：①.增加電容 $C_D = \kappa C_0$，②.結構強度，③.提高**介電強度**(物質不發生介電崩潰可承受之最大電場)。

介電質之高斯定律：$\varepsilon_0 \to \varepsilon = \kappa\varepsilon_0$, $\boldsymbol{D} = \varepsilon\boldsymbol{E}$

$$\varepsilon_0 \oint \boldsymbol{E}\cdot d\boldsymbol{A} = q_{\text{net}} = q_{\text{free}}/\kappa,$$

$$\varepsilon_0 \oint \kappa\boldsymbol{E}\cdot d\boldsymbol{A} = \oint \varepsilon\boldsymbol{E}\cdot d\boldsymbol{A} = q_{\text{free}}。$$

49. (**a**) Initially, the capacitance is $C_0 = \varepsilon_0 A/d$ = 8.85×10^{-12}(0.12)/(1.2×10^{-2}) = 89 (pF). (**b**) Working through *S.P.* 25-7 algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = 1.2\times10^{-10}\text{ (F)} = 120\text{ (pF)}$$

$$= \frac{(8.85\times10^{-12})(0.12)(4.8)}{(4.8)(1.2-0.40)(10^{-2})+4.0\times10^{-3}}.$$

(**c**) Before the insertion, $q = C_0V$ = (89 pF)(120 V) = 11 nC. (**d**) Since the battery is disconnected, q will remain the same after the insertion of the slab, with q = 11 nC. (**e**) $E = q/A\varepsilon_0$ = (11×10^{-9})/(0.12)/(8.85×10^{-12}) = 10 (kV/m). (**f**) $E' = E/\kappa$ = (10 kV/m)/ 4.8 = 2.1 kV/m. (**g**) $V = E(d-b)+E'b$ = (10 kV/m) (0.012 m – 0.0040 m) + (2.1 kV/m)(0.40×10^{-3} m) = 88 V. (**h**) The work done is

$$W_{ext} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(11\times10^{-9})^2}{2}\left(\frac{10^{12}}{89}-\frac{10^{12}}{120}\right)$$

$$= -1.7\times10^{-7}\text{ (J)}.$$

何者造成輪床著火？

S1.電容器<u>用途</u>：高功率電源供應器、急救之電擊器、閃光燈、濾波、調諧器、記憶體等。

S2.在介電質之高斯定律中只出現自由電荷 q_{free}，因其可人為特意地佈置，即可事先得知，因此真空中電場E_0就可易由高斯定律求得，而介電質效應則透過介電常數κ得知。

Flash 快閃/閃存; ferroelectrics 鐵電體(駐電極體); random access memory (RAM)隨機存取記憶體; magnetic recording 磁記錄; •**備忘錄**•

82.• (**a**) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d-b)$ = 0.708 pF. (**b**) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d}$$

$$= \frac{d}{d-b} = \frac{5.00}{5.00-2.00} = 1.67.$$

(**c**) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'}-\frac{1}{C}\right)$$

$$= \frac{q^2}{2\varepsilon_0 A}(d-b-d) = \frac{-q^2 b}{2\varepsilon_0 A} = -5.44\text{ (J)}.$$

(**d**) Since $W < 0$ the slab is sucked in.

☼.平行板電容器: d = 1.00 mm & A = 100 cm^2, 則 C = 8.85×10^{-11} F. 若 C = 1 F & d = 1 mm ⇒ A = 1.13×10^8 m^2, $A^{1/2}$ ~ 10^4 m = 10 km (huge).

☼.兩平行板之 A = 20.0 cm^2 及 d = 0.400 mm, 則 ***a***. C = 4.43× 10^{-11} F; ***b.***若接上 120 V 電池, 則由 $q = CV$ 知 5.32×10^{-9} C 電荷流入電板。

☼.一孤立金屬球(半徑 R)之電容為 $C = 4\pi\varepsilon_0 R$, 若 R = 10.0 cm, C = 1.11×10^{-11} F。

capacitance 電容; farad (F)法拉; capacitor 電容器; plate 電板; battery 電池(組); dielectric strength 介電質強度; dielectric 介電質; polar 極性的; nonpolar 非極性的; polarize 極化; induced charge 感應電荷; electric displacement 電位移; electric circuit 電路; closed(電路)接通的; open(電路)斷的; in series 串聯; in parallel 並聯; gurney 輪床; hyperbaric chamber 高壓處理室; defibrillator 電擊器; burn victim 燒傷患者; Leyden jar 萊頓瓶;

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)