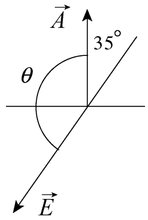
Chapter 23 *Gauss' law*

01. The vector area $\boldsymbol{A}$ and the electric field $\boldsymbol{E}$ are shown on the diagram below. The angle θ between them is 180° – 35° = 145°, so the electric flux through the area is



$\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$
$= (1800\ \text{N/C})(3.2\times10^{-3}\ \text{m})^2\cos145°$
$= -1.5\times10^{-2}\ \text{N}\cdot\text{m}^2/\text{C}$.

07. To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with a proton of charge q = 1.6×10^{-19} C situated at the inside center of the cube. The cube has six faces, and we expect an *equal* amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi_1 = \Phi_{net}/6 = q/6\varepsilon_0 = 3.01\times10^{-9}\ \text{N}\cdot\text{m}^2/\text{C}.$$

09. Let A be the area of one face of the cube, E_u be the magnitude of the electric field at the upper face, and E_ℓ be the magnitude of the field at the lower face. Since the field is downward, the flux through the upper face is negative and the flux through the lower face is positive. The flux through the other faces is zero, so the total flux through the cube surface is $\Phi = A(E_\ell - E_u)$. The net charge inside the cube is given by Gauss' law:

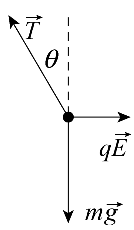
$$q = \varepsilon_0\Phi == \varepsilon_0 A(E_\ell - E_u) = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(100\ \text{m})^2(100\ \text{N/C} - 60.0\ \text{N/C}) = 3.54\times10^{-6}\ \text{C}.$$

16. Using Eq. 23-11, the surface charge density is

$$\sigma = \varepsilon_0 E = (2.3\times10^5\ \text{N/C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 2.0\times10^{-6}\ \text{C/m}^2.$$

32. According to Eq. 23-13 the electric field due to either sheet of charge with surface charge density σ = 1.77×10^{-22} C/m^2 is perpendicular to the plane of the sheet (pointing *away* from the sheet if the charge is positive) and has magnitude $E = \sigma/2\varepsilon_0$. Using the superposition principle, we conclude: (**a**) $E = \sigma/\varepsilon_0 = (1.77\times10^{-22})/(8.85\times10^{-12}) = 2.00\times10^{-11}$ (N/C), pointing in the upward direction, or $\boldsymbol{E}$ = $(2.00\times10^{-11}\ \text{N/C})$**j**. (**b**) $E = 0$; (**c**) and, $E = \sigma/\varepsilon_0$, pointing down, or $\boldsymbol{E} = -(2.00\times10^{-11}\ \text{N/C})$ **j**. (*cf.Pb.35*)

39. The forces acting on the ball are shown in the diagram below. The gravitational force has magnitude mg, where m is the mass of the ball; the electrical force has magnitude qE, where q is the charge on the ball and E is the magnitude of the electric field at the position of the ball; and, the tension in the thread is denoted by T. The electric field produced by the plate is normal to the plate and points to the right. Since the ball is positively charged, the electric force on it also points to the right. The tension in the thread makes the angle θ (= 30°) with the vertical. Since the ball is in equilibrium the net force on it vanishes. The sum of the horizontal components yields $qE - T\sin\theta = 0$ and the sum of the vertical components yields $T\cos\theta - mg = 0$. The expression $T = qE/\sin\theta$, from the first equation, is substituted into the second to obtain $qE = mg\tan\theta$.



The electric field produced by a large uniform plane of charge is given by $E = \sigma/2\varepsilon_0$, where σ is the surface charge density. Thus,

$$\frac{q\sigma}{2\varepsilon_0} = mg\tan\theta$$

and
$$\sigma = \frac{2\varepsilon_0 mg\tan\theta}{q}\quad(= 5.0\times10^{-9}\ \text{C/m}^2)$$
$$= \frac{2(8.85\times10^{-12})(1.0\times10^{-6})(9.80)\tan30°}{2.0\times10^{-8}}.$$

35. We use Eq. 23-13. (**a**) To the left of the plates:

$$\vec{E} = \vec{E}_R\ (\text{from the right plate}) + \vec{E}_L\ (\text{from the left one})$$
$$= \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathrm{i}} = 0.$$

(**b**) To the right of the plates:

$$\vec{E} = \vec{E}_R + \vec{E}_L = \left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathrm{i}} + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) = 0.$$

(**c**) Between the plates:

$$\vec{E} = \vec{E}_R + \vec{E}_L = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) + \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\mathrm{i}}) = \left(\frac{\sigma}{\varepsilon_0}\right)(-\hat{\mathrm{i}})$$
$$= \frac{7.00\times10^{-22}\ \text{C/m}^2}{8.85\times10^{-12}\ \text{N}\cdot\text{m}^2/\text{C}^2}(-\hat{\mathrm{i}}) = -7.91\times10^{-11}\ \text{N/C}\ (\hat{\mathrm{i}}).$$

43. Charge is distributed uniformly over the surface of the sphere and the electric field it produces at points outside the sphere is like the field of a point particle with charge equal to the net charge on the sphere. That is, the magnitude of the field is given by $E = q/4\pi\varepsilon_0 r^2$, where q is the magnitude of the charge on the sphere and r is the distance from the center of the sphere to the point where the field is measured. Thus,

$$q = 4\pi\varepsilon_0 r^2 E = \frac{(0.15)^2(3.0\times10^3)}{8.99\times10^9} = 7.5\times10^{-9}\ \text{(C)}.$$

The field points inward, toward the sphere center, so the charge is negative: -7.5×10^{-9} C.

49. At all points where there is an electric field, it is radially outward. For each part of the problem, use a Gaussian surface in the form of a sphere that is concentric with the sphere of charge and passes through the point where the electric field is to be found. The field is uniform on the surface, so $\int \boldsymbol{E}\cdot d\boldsymbol{A}$ = $4\pi r^2 E$, where r is the radius of the Gaussian surface. For $r < a$, the charge enclosed by the Gaussian surface is $q_1(r/a)^3$. Gauss' law yields

$$4\pi r^2 E = \left(\frac{q_1}{\varepsilon_0}\right)\left(\frac{r}{a}\right)^3 \quad \Rightarrow \quad E = \frac{q_1 r}{4\pi\varepsilon_0 a^3}.$$

(**a**) For $r = 0$, the above equation implies $E = 0$. (**b**) For $r = a/2$, we have

$$E = \frac{q_1 (a/2)}{4\pi\varepsilon_0 a^3} = 8.99\times10^9 \times \frac{5.00\times10^{-15}}{2(2.00\times10^{-2})^2}$$

$$= 5.62\times10^{-2}\ \text{(N/C)}.$$

(**c**) For $r = a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 a^2} = 8.99\times10^9 \times \frac{5.00\times10^{-15}}{(2.00\times10^{-2})^2} = 0.112\ \text{(N/C)}.$$

In the case where $a < r < b$, the charge enclosed by the Gaussian surface is q_1, so Gauss' law leads to

$$4\pi r^2 E = \frac{q_1}{\varepsilon_0} \quad \Rightarrow \quad E = \frac{q_1}{4\pi\varepsilon_0 r^2}.$$

(**d**) For $r = 1.50a$, we have

$$E = \frac{q_1}{4\pi\varepsilon_0 r^2} = 8.99\times10^9 \times \frac{5.00\times10^{-15}}{(1.50\times2.00\times10^{-2})^2}$$

$$= 4.99\times10^{-2}\ \text{(N/C)}.$$

(**e**) In the region $b < r < c$, since the shell is conducting, the electric field is zero. Thus, for $r = 2.30a$, we have $E = 0$. (**f**) For $r > c$, the charge enclosed by the Gaussian surface is zero. Gauss' law yields $4\pi r^2 E = 0 \Rightarrow E = 0$. Thus, $E = 0$ at $r = 3.50a$. (**g**) Consider a Gaussian surface that lies completely within the conducting shell. Since the electric field is everywhere zero on the surface, $\int \boldsymbol{E}\cdot d\boldsymbol{A} = 0$ and, according to Gauss' law, the net charge enclosed by the surface is zero. If Q_i is the charge on the inner surface of the shell, then $q_1+Q_i = 0$ and $Q_i = -q_1 = -5.00$ fC. (**h**) Let Q_o be the charge on the outer surface of the shell. Since the net charge on the shell is $-q$, $Q_i+Q_o = -q_1$. This means $Q_o = -q_1 - Q_i = -q_1-(-q_1) = 0$.

70. Since the fields involved are uniform, the precise location of P is not relevant; what is important is it is above the three sheets, with the positively charged sheets contributing upward fields and the negatively charged sheet contributing a downward field, which conveniently conforms to usual conventions (of upward as positive and downward as negative). The net field is directed upward (+**j**), and (from Eq. 23-13) its magnitude is

$$|\vec{E}| = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} = \frac{1.0\times10^{-6}}{2\times8.85\times10^{-12}} = 5.65\times10^4 \text{(N/C)}.$$

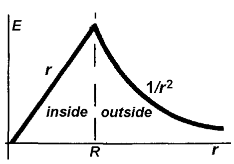
In unit-vector notation, we have

$$\vec{E} = (5.65\times10^4\ \text{N/C})\,\hat{\mathrm{j}}.$$

72. (**a**) From Gauss' law,

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q_{encl}}{r^3}\vec{r}$$

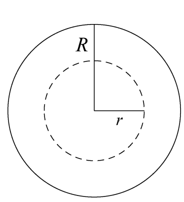
$$= \frac{1}{4\pi\varepsilon_0}\frac{(4\pi\rho r^3/3)}{r^3}\vec{r} = \frac{\rho\vec{r}}{3\varepsilon_0}.$$



(**b**)* The charge distribution in this case is equivalent to that of a whole sphere of charge density ρ plus a smaller sphere of charge density $-\rho$ which fills the void. By superposition

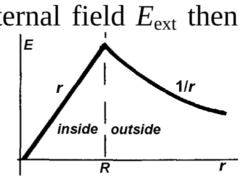
$$\vec{E}(\vec{r}) = \frac{\rho\vec{r}}{3\varepsilon_0} + \frac{(-\rho)(\vec{r}-\vec{a})}{3\varepsilon_0} = \frac{\rho\vec{a}}{3\varepsilon_0}.$$

85. (**a**) The diagram below shows a cross section (or, perhaps more appropriately, "end view") of the charged cylinder (solid circle). Consider a Gaussian surface in the form of a cylinder with radius r and length ℓ coaxial with the charged cylinder. An "end view" of the Gaussian surface is shown as a dotted circle. The charge enclosed by it is $q = \rho V = \pi r^2 \ell\rho$, where $V = \pi r^2 \ell$ is the volume of the cylinder. If ρ is positive, the electric field lines are radially outward, normal to the Gaussian surface and distributed uniformly along it. Thus, the total flux through the Gaussian cylinder is $\Phi = EA_{\text{cylinder}} = E(2\pi r\ell)$. Now, Gauss' law leads to



$$2\pi\varepsilon_0 r\ell E = \pi r^2 \ell\rho \Rightarrow E = \frac{\rho r}{2\varepsilon_0}.$$

(**b**) Next, we consider a cylindrical Gaussian surface of radius $r > R$. If the external field E_{ext} then the flux is $\Phi = E_{\text{ext}}(2\pi r\ell)$. The charge enclosed is the total charge in a section of the charged cylinder with length ℓ. That is, $q = \pi R^2 \ell\rho$. In this case, Gauss' law yields



$$2\pi\varepsilon_0 r\ell E_{\text{ext}} = \pi R^2 \ell\rho \Rightarrow E_{\text{ext}} = \frac{R^2\rho}{2\varepsilon_0 r}.$$

34.* The charge distribution in this problem is equivalent to that of an infinite sheet of charge with surface charge density $\sigma = 4.5\times10^{-12}$ C/m^2 plus a small circular pad of radius $R = 1.80$ cm located at the middle of the sheet with charge density $-\sigma$. We denote the electric fields produced by the sheet and the pad with subscripts 1 and 2, respectively. Using Eq. 22-26 for E_2, the net electric field E at a distance $z = 2.56$ cm along the central axis is then

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \hat{\mathrm{k}}\left(\frac{\sigma}{2\varepsilon_0}\right) + \hat{\mathrm{k}}\left(\frac{-\sigma}{2\varepsilon_0}\right)\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

$$= \frac{\sigma z}{2\varepsilon_0\sqrt{z^2+R^2}}\,\hat{\mathrm{k}} = (0.208\ \text{N/C})\,\hat{\mathrm{k}}.$$

flux 流量/通量;electric flux 電通量;Gauss' law 高斯定律; spherical/cylindrical/planar symmetry 球/圓柱/平面對稱; Gaussian surface 高斯曲面; electrostatic equilibrium/shielding 靜電平衡/屏蔽; cavity 空腔; lightning 閃電; side flash 側閃電; upward streamer 向上電流; lookout platform 觀景台; Sequoia National Park 美洲杉國家公園(內有世上直徑最大之樹--美洲杉基部的直徑 11.1m, 周圍 31.1m,高度 83.8m,樹齡 2000yr)

重點整理－第 23 章 高斯定律

體流率(水流率)：單位時間通過某截面之流體體積，即流速與截面面積向量之純量乘積(內積)。

電通量：電場與面積向量之純量乘積(內積)。

高斯定律：通過任一封閉曲面的電通量等於該封閉曲面所包圍的淨電荷 $q_{\rm enc}$ 除以 ε_0，

$$\oint \vec{E}\cdot d\vec{A} = q_{\rm enc}/\varepsilon_0 \text{ or } \varepsilon_0\oint \vec{E}\cdot d\vec{A} = q_{\rm enc}。$$

Note **高斯定律**描述空間電場的特性，而**庫倫定律**描述兩點電荷間作用力性質，但兩者為全等的。

Note 點電荷 q 所產生之電場通過(***a***)包圍其的球面之電通量 $\Phi = q/\varepsilon_0$; (***b***)其旁之無限大平面的電通量 $\Phi = q/2\varepsilon_0$。(***c***)點電荷 q 放於立方體之中心，通過各面之電通量 $\Phi = q/6\varepsilon_0$ (*cf. CP*. 2 & *Pb*.7 & 55)。(***d***)電荷 q 放於正方形(邊長 a)之中心正上方距離 $a/2$ 處，通過此面之電通量 $\Phi =$ ___。

閃電風暴之另外危險為何？

作業 **3-1**.(***a***)電荷 q 放於正方形(邊長 a)之頂點正上方距離 a 處，通過此面之電通量。(***b***)點電荷 q 放於長方形(邊長 a 及 $a/2$)之長邊中心正上方 $a/2$ 處，通過此面之電通量。

在電場中之導體：於靜電平衡時，***A***.導體內部的電場為零。***B***.導體內部沒有多餘(淨)電荷；如有多餘電荷必留駐於導體表面。***C***.導體表面處的電場垂直於導體表面，其電場大小 $E_s = \sigma/\varepsilon_0$，$\sigma$為表面電荷密度。**靜電屏蔽**：一密閉之導體空腔，可將電場完全排除於外，即屏蔽電場線。***Note*** 當電荷佈置於非導體內時，電荷就不再移動。

運用高斯定律—具對稱性電荷分佈之電場：

1.點電荷(球對稱電荷分佈) $E = q/4\pi\varepsilon_0 r^{-2} \propto r^{-2}$,
非導體球內電荷均勻分佈之電場(*cf. Pb.72*)

2.均勻線電荷(圓柱對稱電荷)分佈

$$E = \lambda/2\pi\varepsilon_0 r \propto r^{-1},$$

非導體圓柱內電荷均勻分佈之電場(*cf. Pb.85*)

3.非導體面上均勻電荷分佈 $E = \sigma/2\varepsilon_0 \propto r^0$,

4.導體面上均勻電荷分佈 $E = \sigma/\varepsilon_0 \propto r^0$,

Note 電偶極之遠場 $E \propto r^{-3}$,

作業 **3-2**.某導體球殼[內(外)半徑為 a (b)]之淨帶電量為$-2q$ ($q > 0$)，其殼心處放一點電荷$+3q$，試計算(***a***)空間各點(至殼心的距離為 r)之電場，(***b***)球殼內外表面之電荷密度。 •**備忘錄**•

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)