Chapter ***16*** ***Waves – I***

01. (**a**) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is T = 4(0.170 s) = 0.680 s. (**b**) The frequency is the reciprocal of the period: f = 1/T = 1/0.680 s = 1.47 Hz. (**c**) A sinusoidal wave travels one wavelength in one period: $v = \lambda/T$ = 1.40 m/0.680 s = 2.06 m/s.

06. (**a**) The amplitude is y_m = 6.0 cm. (**b**) We find λ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm. (**c**) Solving $2\pi f = \omega = 4.0\pi$, we obtain f = 2.0 Hz. (**d**) The wave speed is $v = \lambda f$ = (100 cm) (2.0 Hz) = 2.0×10^2 cm/s. (**e**) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2). (**f**) The maximum transverse speed (found from the time derivative of y) is $u_{\max} = 2\pi f y_{\max} = (4.0\pi\ \mathrm{s}^{-1})(6.0\ \mathrm{cm})$ = 75 cm/s. (**g**) y(3.5 cm, 0.26 s) = (6.0 cm) sin[0.020π (3.5)+4.0π (0.26)] = –2.0 cm.

09. (**a**) The amplitude y_m is half of the 6.0 mm vertical range shown in the figure, i.e., y_m = 3.0 mm . (**b**) The speed of the wave is $v = d/t$ = 15 m/s, where d = 0.060 m and t = 0.0040 s. The angular wave number is $k = 2\pi/\lambda$ where λ = 0.40 m. Thus, $k = 2\pi/\lambda$ = 16 rad/m. (**c**) The angular frequency is found from $\omega = kv$ = (16)(15) = 2.4×10^2 (rad/s). (**d**) We choose the minus sign (between kx and ωt) in the argument of the sine function because the wave is shown traveling to the right [in the +x direction] – see §16-5). Therefore, with SI units under- stood, we obtain

$$y = y_m \sin(kx-\omega t) \approx 0.0030 \sin(16x-2.4\times10^2\, t).$$

12. The volume of a cylinder of height ℓ is $V = \pi r^2\ell = (¼)\pi d^2\ell$. The strings are long, narrow cylinders, one of diameter d_1 and the other of diameter d_2 (and corresponding linear densities μ_1 and μ_2). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is $\mu = m/\ell = \rho (¼)(\pi d^2 \ell)/\ell = (¼)\pi\rho d^2$ and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho d_1^2/4}{\pi\rho d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2.$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

15. (**a**) The wave speed is given by $v = \lambda/T = \omega/k$, where λ is the wavelength, T is the period, ω (= $2\pi/T$)is the angular frequency, and k (= $2\pi/\lambda$) is the angular wave number. The displacement has the form $y = y_m \sin(kx+\omega t)$, so $k = 2.0\ \mathrm{m}^{-1}$ and ω = 30 rad/s. Thus $v = (30\ \mathrm{rad/s})/(2.0\ \mathrm{m}^{-1})$ = 15 m/s. (**b**) Since the wave speed is given by $v = (\tau/\mu)^{1/2}$, where τ is the tension in the string and μ is the linear mass density of the string, the tension is $\tau = \mu v^2 = (1.6\times10^{-4}\ \mathrm{kg/m})(15\ \mathrm{m/s})^2$ = 0.036 N.

24. Using Eq. 16-33 for the average power and Eq. 16-26 for the speed of the wave, we solve for $f = \omega/2\pi$,

$$f^2 = \frac{1}{4\pi^2 y_m^2}\,\frac{2P_{av}}{\mu\sqrt{\tau/\nu}} = \frac{1}{4\pi^2(7.70x10^{-3})^2}$$

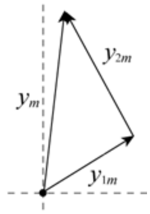
$$\times\frac{2(85.0)}{\sqrt{(36.0)(0.260/2.70)}} \Rightarrow f = 198\ \mathrm{Hz}.$$

30. (**a**) Let the phase difference be ϕ. Then from Eq. 16–52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}(1.50y_m/2y_m) = 82.8°.$$

(**b**) Converting to radians, we have ϕ = 1.45 rad. (**c**) In terms of wavelength (the length of each cycle, where each cycle corresponds to 2π rad), this is equivalent to 1.45 rad/2π = 0.230 wavelength.

33. The phasor diagram is shown below: y_{1m} and y_{2m} represent the original waves and y_m represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives



$$y_m^2 = y_{1m}^2 + y_{1m}^2 = (3\ \mathrm{cm})^2 + (4\ \mathrm{cm})^2 = (25\ \mathrm{cm})^2.$$

Thus y_m = 5.0 cm.

37. (**a**) Using the phasor technique, we think of these as two "vectors" (the first of "length" 4.6 mm and the second of "length" 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then leads to a resultant vector of length 3.29 mm. (**b**) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad). (**c**) Clearly, it should in "in phase" with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

41. (**a**) The wave speed is given by $v = (\tau/\mu)^{1/2}$ where τ is the tension in the string and μ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where M is the mass of the string and L is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0)(8.40)}{0.120}} = 82.0\ \mathrm{(m/s)}.$$

(**b**) The longest possible wavelength λ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L$ = 2(8.40 m) = 16.8 m. (**c**) The frequency is $f = v/\lambda$ = (82.0)/(16.8) = 4.88 (Hz).

43. (**a**) Eq. 16–26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150}{7.20\times10^{-3}}}$$

$$= 144.34\ \mathrm{(m/s)} \approx 1.44\times10^2\ \mathrm{(m/s)}.$$

(**b**) From the Figure, we find the wavelength of the standing wave to be λ = (2/3)(90.0 cm) = 60.0 cm. (**c**) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times 100}{0.600} = 241 \text{ (Hz)}.$$

46. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus, f_1 = 390 Hz – 325 Hz = 65 Hz. This further implies that the next higher resonance above 195 Hz should be 195 Hz + 65 Hz = 260 Hz.

72.• (**a**) With length in centimeters and time in seconds, we have

$$u = dy/dt = -60\pi\cos(\frac{\pi}{8}x - 4\pi t).$$

Thus, when x = 6 cm and t = 1/4 s, we obtain

$$u = -60\pi\cos\frac{-\pi}{4} = -\frac{60\pi}{\sqrt{2}} = -133,$$

so that the *speed* there is 1.33 m/s. (**b**) The numerical coefficient of the cosine in the expression for u is -60π. Thus, the maximum *speed* is 1.88 m/s. (**c**) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2\sin(\frac{\pi}{8}x - 4\pi t),$$

so that when x = 6 cm and t = 1/4 s we obtain $a = -240\pi^2\sin(\pi/4)$ which yields a = 16.7 m/s^2. (**d**) The numerical coefficient of the sine in the expression for a is $-240\pi^2$. Thus, the maximum acceleration is 23.7 m/s^2.

79.• We use Eqs. 16-2, 16-5, 16-9, 16-13, and take the derivative to obtain the transverse speed u. (**a**) The amplitude is y_m = 2.0 mm. (**b**) Since ω = 600 rad/s, the frequency is found to be $f = 600/2\pi \approx 95$ Hz. (**c**) Since k = 20 rad/m, the velocity of the wave is $v = \omega/k$ = 600/20 = 30 m/s in the +x direction. (**d**) The wavelength is $\lambda = 2\omega/k \approx 0.31$ m, or 31 cm. (**e**) We obtain

$$u = dy/dt = -\omega y_m\cos(kx-\omega t) \Rightarrow u_m = \omega y_m,$$

so that the maximum transverse speed is u_m = (600)(2.0) = 1200 (mm/s), or 1.2 m/s.

82.• (**a**) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where λ is the wavelength and L is the length of the string. The wavelength is related to the frequency f and wave speed v by $\lambda = v/f$, so $L/2 = v/f$ and $L = 2v/f$ = 2(400 m/s)/(600 Hz) = 1.3 m. (**b**) The expression for the string displacement is $y = y_m\sin(kx)\cos(\omega t)$, where y_m is the maximum displacement, k is the angular wave number, and ω is the angular frequency $k = 2\pi/\lambda = 2\pi f/v = 2\pi$ (600 Hz)/(400 m/s) = 9.4 m^{-1}, $\omega = 2\pi f = 2\pi$ (600 Hz) = 3800 rad/s, y_m = 2.0 mm. The displacement is given by

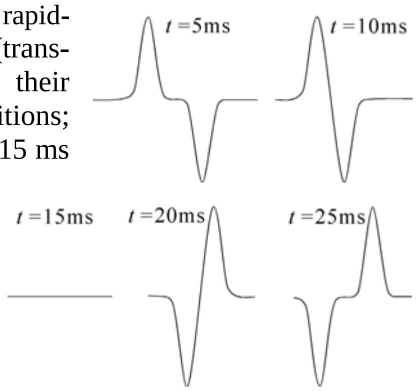
$$y(x, t) = (2.0 \text{ mm})\sin[(9.4 \text{ m}^{-1})x]\cos[(3800 \text{ s}^{-1})t].$$

86.• (**a**) Let the displacements of the wave at (y, t) be $z(y, t)$. Then $z(y, t) = z_m\sin(ky-\omega t)$, where z_m = 3.0 mm, k = 60 cm^{-1}, and $\omega = 2\pi/T = 2\pi/0.20$ s = 10π s^{-1}. Thus

$$z(y, t) = z_m\sin[(60 \text{ m}^{-1})y - (10\pi \text{ s}^{-1})t].$$

(**b**) The maximum transverse speed is $u_m = \omega z_m$ = (2π/0.20 s)(3.0 mm) = 94 mm/s.

93.• (**a**) We note that each pulse travels 1 cm during each Δt = 5 ms interval. Thus, in these first two pictures, their peaks are closer to each other by 2 cm, successively. And the next pictures show the (momentary) complete cancellation of the visible pattern at t = 15 ms, and the pulses moving away from each other after that. (**b**) The particles of the string are moving rapidly as they pass (transversely) through their equilibrium positions; the energy at t = 15 ms is purely kinetic.



Ex.1-1, *Pb. 16-45*.

23.* (**a**) The wave speed at any point on the rope is given by $v = (\tau/\mu)^{1/2}$, where τ is the tension at that point and μ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance y from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by μgy, so the tension is $\tau = \mu gy$. The wave speed is $v = (\mu gy/\mu)^{1/2} = (gy)^{1/2}$. (**b**) The time dt for the wave to move past a length dy, a distance y from the bottom end, is $dt = dy/v = dy/(gy)^{1/2}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\Big|_0^L = 2\sqrt{\frac{L}{g}}.$$

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

S1. $\sin\alpha + \sin\beta = 2\sin\frac{1}{2}(\alpha+\beta)\cos\frac{1}{2}(\alpha-\beta)$.

S2. time or spatial average of sine functions

$$\langle\sin(kx\pm\omega t)\rangle = 0, \quad \langle\sin^2(kx\pm\omega t)\rangle = \frac{1}{2}$$ 。

S3. $y(x, t) = y_m\sin(kx\pm\omega t)$,

$\partial y/\partial t = \pm\omega y_m\cos(kx\pm\omega t)$, $\partial^2 y/\partial t^2 = \omega^2 y_m\sin(kx\pm\omega t)$

$\partial y/\partial x = k\, y_m\cos(kx\pm\omega t)$, $\partial^2 y/\partial x^2 = k^2 y_m\sin(kx\pm\omega t)$ 。

重點整理－第 16 章　波 動 I

波動為偏離平衡條件的擾動(之現象)，其能隨著時間進展從空間的一處行進或傳播到另一處。<u>波動能傳播**能量及動量**</u>，<u>但介質內質點並不隨之傳播出去</u>。我們生活世界充滿波動，波動帶給我們**有聲有色、多采多姿的生活**！波動現象的例子俯拾即是。波依其本質可分類成三大類：**1**.力學波，**2**.電磁波與 **3**.物質波—"基本粒子"的波似行為。

♦**力學波**：藉物質(介質)之形變而傳播的波動，如弦波、聲波、水波及地震波等。其主要特點為(a)遵循牛頓力學，(b)須於**介質**內才能存在。

依介質振動方式，可分為"**橫波**"與"**縱波**"兩類。

橫波：介質內質點的位移與波行進的方向**垂直**。

縱波：介質內質點之運動方向與波行進的方向平行。　　♦脈衝波→ 連續波→ 諧和波

假設弦為(**a**)無阻尼及(**b**)無限長。

波函數：描述介質內各質點於任一時間的位移(位置)之函數。 $h(x, t)$包含一完整的波運描述。

$$y = h(x, t) = h(kx \pm \omega t) = h(x \pm vt) \equiv y(x, t),$$

諧和(正弦)波：波函數爲諧和函數之波動或波源作簡諧運動之波動，當諧和波通過介質時，各質點以相同的週期但特定的相位差作簡諧運動。

$$y(x, t) = y_m \sin(kx \pm \omega t) = y_m \sin 2\pi\left(\frac{x}{\lambda} \pm \frac{t}{T}\right),$$

振幅 y_m：弦元最大位移；**波長**λ：波形上兩連續極大或極小的距離，或介質內具有相同相位之兩點的最短距離；**角波數** $k = 2\pi/\lambda$；**週期**$= T$；**頻率** $f\,(= 1/T)$；**角頻率** $\omega = 2\pi f$；–號: → or +號: ←。

波速 v：波形傳播的速率 $v = \omega/k = \lambda/T = \lambda f$，

"**傳播速率 v 等於頻率 f 與波長λ的乘積**" 。

Note (**a**)"**波形的傳播速率**"與"**波行經介質時其內質點的運動速率**"之區別。(**b**)波速由介質之力學性質決定，當波頻率改變，波長隨之而變，但波速不變。

波速(張緊弦上之波)：$v = \sqrt{\tau/\mu}\ (= \lambda f)$，

$\tau \equiv$ 張力(N)，$\mu \equiv$ 線質量密度(kg/m)。

傳輸平均功率(張緊弦上諧和波) 弦元運動，具動能；再者弦上各處變形不一致，而儲存位能，因此**傳輸功率**　$P = dE/dt = dK/dt + dU/dt,$

$$P_{av} = \frac{1}{2}\mu v \omega^2 y_m^2。$$

波動方程式 $\partial^2 y/\partial t^2 = v^2\, \partial^2 y/\partial x^2$。

波之疊加原理：介質內質點在任一時刻之實際位移可藉著將各波單獨存在時具有的位移相加(向量和)而得到。♦**干涉**：兩個或多個波同時通過相同區域造成的合成現象。

合成波或**淨波**：設兩波具有相同的振幅及波長或頻率，但兩波具相位差/相位偏移 $\phi: -\pi \sim \pi$,

$$z(x, t) = y_1(x, t) + y_2(x, t) = y_m \sin(kx - \omega t)$$
$$+ y_m \sin(kx - \omega t + \phi) = [2y_m \cos(\tfrac{1}{2}\phi)]\sin(kx - \omega t + \tfrac{1}{2}\phi),$$

兩波之相位差/相位偏移 $\phi: -\pi \sim \pi$,

合成波振幅 $z_m = 2y_m \cos\frac{1}{2}\phi$，與相位差有關。

(a) $\phi = 0$，完全建設性干涉，$z_m = 2y_m$; (b) $\phi = \pi$，完全破壞性干涉，$z_m = 0$; (c) 其它ϕ，中級干涉。

相量：具相位之量，運算時類似向量，於(二維)相量圖上，向量大小(長度)爲波的"振幅"而向量夾角爲波的"相位"，兩相量之合成如同向量般。

弦上駐波：介質內如同時通過兩個頻率及振幅相同但行進方向相反之波時，其產生駐波。

向右之波 $y_1(x, t) = y_m \sin(kx - \omega t)$，

向左之波 $y_2(x, t) = y_m \sin(kx + \omega t)$。 合成波 ⇒

駐波 $z(x, t) = y_1(x, t) + y_2(x, t) = (2y_m \sin kx)\cos\omega t$。

駐波特徵：a)合成波之波形不行進，弦上質點只於原先平衡處漲伏起落；b)振幅 $z_m = 2y_m|\sin kx|$，隨位置而改變；c)節點—永不動之點，振幅 $z_m = 0$；d)腹點或反節點—振幅最大之點 $z_m = 2y_m$。

相鄰兩波節(腹)的距離為半波長($\lambda/2$)。

波於弦邊界(端點)之**反射**：反射波反向行進，而反射波與入射波之相位差爲(**a**)π ，當端點爲固定的，或(**b**) 0 (無相位差)，當端點爲自由的。

共振頻率為基頻(最小共振頻率 f_1)之整數倍(有限長度 L 且兩端固定的弦，只有特定波長之駐波存在)

$$f_n = n f_1, \text{ for } n = 1, 2, 3, \ldots; \quad f_1 = \frac{v}{2L},\ v = \sqrt{\frac{\tau}{\mu}};$$

式中 $n =$ 諧音數或模數。

弦樂器基本發音原理：

弦短、**質量密度小**、**張力大** ⇒ **基頻高**。

泛音：頻率高於基頻之諧音。

遙遠地方的測量如何決定潛艇的深度？

wave 波/波動; transverse / longitudinal wave 橫/縱波; mechanic/electromagnetic/matter wave 力學/電磁/物質波; seismic wave 地震波; traveling wave 行進波; wave form 波形; wavelength 波長; period 週期; medium 介質; phasor 相量; vector 向量; constructive/intermediate/destructive interference 建設性/中級/破壞性干涉; resultant /net wave 合成/淨波; in phase 同相; out of phase 異相; node 節點; antinode 腹/反節點; disturbance 擾動; wave pulse 波脈衝; periodic wave 週期波; harmonic 諧和的; sinusoidal 正弦的; wave speed 波速; wave function 波函數; wave equation 波動方程式; superposition principle 波之疊加原理;fixed 固定的;free 自由的; Kursk 庫爾斯克(地名,潛艇); Barents Sea 巴倫支海; bizarre 奇異的; oscillation mode 振盪模式(standing wave pattern 駐波圖案); first harmonic 第一諧音(波); harmonic number 諧音數; loop 圈; harmonic series 諧音系列; overtone 泛音; piano 鋼琴, guitar 吉它, violin 小提琴, viola 中提琴, Cello 大提琴, Bass 低音提琴; kettledrum 定音鼓, snare drum 響弦鼓; drumhead 鼓膜;

S4. 數學上**波之疊加原理**可成立的原因是因爲波方程式式是線性的，因此假如 $y_1(x, t)$與 $y_2(x, t)$爲波動方程式的解，則 $y_1(x, t)+y_2(x, t)$亦是其解，設質點振動面相同時。 $L(\alpha x+\beta y) = \alpha L(x) + \beta L(y)$，稱 L 是線性的。

S5. 弦波之(彈性)位能表示：平衡時之某弦元長 dx (水平), 波通過時之此弦元長變為 $d\ell$ (傾斜), 因此張力所作的功為彈性位能 $dU = \tau(d\ell - dx)$, $d\ell = [(dx)^2+(dy)^2]^{1/2}$, 對小振幅波動,

$d\ell = dx[1+(\partial y/\partial x)^2]^{1/2} \approx dx[1+(½)(\partial y/\partial x)^2+\ldots]$,

因此得 $dU = (½)\tau(\partial y/\partial x)^2\, dx$.

若 $y(x, t) = y_m \sin(kx-\omega t)$, $\partial y/\partial x = k\, y_m \cos(kx-\omega t)$,

得 $dU = (½)\tau k^2 y_m^2 \cos^2(kx-\omega t)\, dx$, $(\tau k^2 = \mu\omega^2)$,

$$dU/dt = (½)\, v\tau k^2 y_m^2 \cos^2(kx-\omega t) = dK/dt.$$

Pb.17-4, 5, 7, 8, 16, 19, 23, 26, 30, 38, 42, 43, 45, 47, 51, 54, 57, 63 (*tentatively*) •**備忘錄**•