## Chapter 14 Fluids

01．The air inside pushes outward with a force given by $p_{i} A$ ，where $p_{i}$ is the pressure inside the room and $A$ is the area of the window．Similarly，the air on the outside pushes inward with a force given by $p_{0} A$ ，where $p_{0}$ is the pressure outside．The magnitude of the net force is $F=\left(p_{i}-p_{o}\right) A$ ．Since 1 $\mathrm{atm}=1.013 \times 10^{5} \mathrm{~Pa}, F=(1.0 \mathrm{~atm}-0.96 \mathrm{~atm})$ $\left(1.013 \times 10^{5} \mathrm{~Pa} / \mathrm{atm}\right)(3.4 \mathrm{~m})(2.1 \mathrm{~m})=2.9 \times 10^{4} \mathrm{~N}$ ．
05．Let the volume of the expanded air sacs be $V_{a}$ and that of the fish with its air sacs collapsed be $V$ ． Then $\rho_{\text {fish }}=m_{\text {fish }} / V=1.08 \mathrm{~g} / \mathrm{cm}^{3}$ and $\rho_{w}=m_{\text {fish }} /$ $\left(V+V_{a}\right)=1.00 \mathrm{~g} / \mathrm{cm}^{3}$ ，where $\rho_{w}$ is the density of the water．This implies $\rho_{\text {fish }} V=\rho_{w}\left(V+V_{a}\right)$ or $\left(V+V_{a}\right) / V$ $=1.08 / 1.00$ ，which gives $\quad V_{d} /\left(V+V_{a}\right)=7.4 \%$ ．
07．（a）The pressure difference results in forces applied as shown in the figure．We consider a team of horses pulling to the right．To pull the sphere apart，the team must exert a force at least as great as the horizontal component of the total force deter－ mined by＂summing＂（actually，integrating）these force vectors．We consider a force vector at angle $\theta$ ． Its leftward component is $\Delta p \cos \theta d A$ ，where $d A$ is the area element for where the force is applied．We make use of the symmetry of the problem and let $d A$ be that of a ring of constant $\theta$ on the surface． The radius of the ring is $r=R \sin \theta$ ，where $R$ is the radius of the sphere．If the angular width of the ring is $d \theta$ ，in radians，then its width is $R d \theta$ and its area is $d A=2 \pi R^{2} \sin \theta d \theta$ ．Thus the net horizontal compo－ nent of the force of the air is given by

$$
\begin{aligned}
F & =2 \pi R^{2} \Delta p \int_{0}^{\pi / 2} \sin \theta \cos \theta d \theta \\
& =\left.\pi R^{2} \Delta p \sin ^{2} \theta\right|_{0} ^{\pi / 2}=\pi R^{2} \Delta p .
\end{aligned}
$$

（b）We use $1 \mathrm{~atm}=1.01 \times 10^{5}$ Pa to show that $\Delta p=$ $0.90 \mathrm{~atm}=9.09 \times 10^{4} \mathrm{~Pa}$ ．The sphere radius is $R=$ 0.30 m ，so $F_{h}=\pi(0.30 \mathrm{~m})^{2}\left(9.09 \times 10^{4} \mathrm{~Pa}\right)=2.6 \times 10^{4}$ N ．（c）One team of horses could be used if one half of the sphere is attached to a sturdy wall．The force of the wall on the sphere would balance the force of the horses．
11．The pressure $p$ at the depth $d$ of the hatch cover is $p_{0}+\rho g d$ ，where $\rho$ is the density of ocean water and $p_{0}$ is atmospheric pressure．The downward force of the water on the hatch cover is $\left(p_{0}+\rho g d\right) A$ ， where $A$ is the area of the cover．If the air in the submarine is at atmospheric pressure then it exerts an upward force of $p_{0} A$ ．The minimum force that must be applied by the crew to open the cover has magnitude

$$
\begin{aligned}
& F=\left(p_{0}+\rho g d\right) A-p_{0} A=\rho g d A \\
& =\left(1024 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(100 \mathrm{~m})(1.2 \mathrm{~m})(0.60 \mathrm{~m}) \\
& =7.2 \times 10^{5} \mathrm{~N} .
\end{aligned}
$$

08．Note that 0.05 atm equals $5065 \mathrm{~N} / \mathrm{m}^{2}$ ．Applica－ tion of Eq．14－7 with the notation in this prob．leads
to $d_{\max }=5065 / \rho_{\text {liquid }} g$ ，with SI units understood． Thus the difference of this quantity between fresh water（ $998 \mathrm{~kg} / \mathrm{m}^{3}$ ）and Dead Sea water $\left(1500 \mathrm{~kg} / \mathrm{m}^{3}\right)$ is

$$
d_{\max }=\frac{5065}{9.8}\left(\frac{1}{998}-\frac{1}{1500}\right)=0.17(\mathrm{~m})
$$

15．When the levels are the same the height of the liquid is $h=\left(h_{1}+h_{2}\right) / 2$ ，where $h_{1}$ and $h_{2}$ are the original heights．Suppose $h_{1}$ is greater than $h_{2}$ ．The final situation can then be achieved by taking liquid with volume $A\left(h_{1}-h\right)$ and mass $\rho A\left(h_{1}-h\right)$ ，in the first vessel，and lowering it a distance $h-h_{2}$ ．The work done by the force of gravity is $W=\rho A\left(h_{1}-h\right) g$ $\left(h-h_{2}\right)$ ．We substitute $h=\left(h_{1}+h_{2}\right) / 2$ to obtain

$$
W=(1 / 4) \rho g A\left(h_{1}-h_{2}\right)^{2}
$$

$=(1 / 4)\left(1.30 \times 10^{3}\right)(9.80)\left(4.00 \times 10^{-4}\right)(1.56-0.854)^{2}$
$=0.635(\mathrm{~J})$ ．
19．（a）At depth $y$ the gauge pressure of the water is $p=\rho g y$ ，where $\rho$ is the density of the water．We consider a horizontal strip of width $W$ at depth $y$ ， with（vertical）thickness $d y$ ，across the dam．Its area is $d A=W d y$ and the force it exerts on the dam is $d F$ $=p d A=\rho g y W d y$ ．The total force of the water on the dam is

$$
F=\int_{0}^{D} \rho g W y d y=\frac{1}{2} \rho g W D^{2}=\frac{1}{2}\left(1.00 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right)
$$

$$
\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(314 \mathrm{~m})(35.0 \mathrm{~m})^{2}=1.88 \times 10^{9} \mathrm{~N}
$$

（b）Again we consider the strip of water at depth $y$ ． Its moment arm for the torque it exerts about $O$ is $D-y$ so the torque it exerts is $d \tau=d F(D-y)=$ $\rho g y W(D-y) d y$ and the total torque of the water is

$$
\begin{aligned}
& \tau=\int_{0}^{D} \rho g y W(D-y) d y=\rho g W\left(\frac{1}{2}-\frac{1}{3}\right) D^{3}=\frac{1}{6} \rho g W D^{3} \\
& =\frac{1}{6}\left(1.00 \times 10^{3}\right)(9.80)(314)(35.0)^{3}=2.20 \times 10^{10} \mathrm{~N} \cdot \mathrm{~m}
\end{aligned}
$$

（c）We write $\tau=r F$ ，where $r$ is the effective moment arm．Then，

$$
r=\frac{\tau}{F}=\frac{\rho g W D^{3} / 6}{\rho g W D^{2} / 2}=\frac{D}{3}=\frac{35.0}{3}=11.7(\mathrm{~m}) .
$$

20．The gauge pressure you can produce is

$$
\begin{gathered}
p=-\rho g h=-(1000)(9.8)\left(4.0 \times 10^{-2}\right) /\left(1.01 \times 10^{5}\right) \\
=-3.9 \times 10^{-3}(\mathrm{~atm}),
\end{gathered}
$$

where the minus sign indicates that the pressure inside your lung is less than the outside pressure．
22．（a）According to Pascal＇s principle $F / A=f / a \Rightarrow$ $F=(A / a) f$ ．（b）We obtain

$$
f=\frac{a}{A} F=\frac{(3.80 \mathrm{~cm})^{2}}{(53.0 \mathrm{~cm})^{2}}\left(20.0 \times 10^{3} \mathrm{~N}\right)=103 \mathrm{~N}
$$

The ratio of the squares of diameters is equivalent to the ratio of the areas．We also note that the area units cancel．
26．（a）The pressure（including the contribution from the atmosphere）at a depth of $h_{\text {top }}=L / 2$（corresponding
to the top of the block）is

$$
\begin{aligned}
p_{\text {top }}=p_{\text {atm }}+\rho g h_{\text {top }} & =\left[1.01 \times 10^{5}+(1030)(9.8)(0.300)\right] \\
& =1.04 \times 10^{5}(\mathrm{~Pa}),
\end{aligned}
$$

where the unit Pa （Pascal）is equivalent to $\mathrm{N} / \mathrm{m}^{2}$ ． The force on the top surface（of area $A=L^{2}=0.36$ $\mathrm{m}^{2}$ ）is $F_{\text {top }}=p_{\text {top }} A=3.75 \times 10^{4} \mathrm{~N}$ ．（b）The pressure at a depth of $h_{\mathrm{bot}}=3 L / 2$（that of the bottom of the block）
is $\quad p_{\text {bot }}=p_{\text {atm }}+\rho g h_{\text {bot }}$
$=\left[1.01 \times 10^{5}+(1030)(9.8)(0.900)\right]=1.10 \times 10^{5}(\mathrm{~Pa})$ ， where we recall that the unit Pa （Pascal）is equiva－ lent to $\mathrm{N} / \mathrm{m}^{2}$ ．The force on the bottom surface is $F_{\text {bot }}$ $=p_{\text {bot }} A=3.96 \times 10^{4} \mathrm{~N}$ ．（c）Taking the difference $F_{\text {bot }}-F_{\text {top }}$ cancels the contribution from the atmo－ sphere（including any numerical uncertainties associated with that value）and leads to

$$
F_{\text {bot }}-F_{\text {top }}=\rho g\left(h_{\text {bot }}-h_{\text {top }}\right) A=\rho g L^{3}=2.18 \times 10^{3}(\mathrm{~N}),
$$

which is to be expected on the basis of Archimedes’ principle．Two other forces act on the block：an up－ ward tension $T$ and a downward pull of gravity mg ． To remain stationary，the tension must be

$$
\begin{aligned}
T= & m g-\left(F_{\text {bot }}-F_{\text {top }}\right)=(450)(9.80) \\
& -2.18 \times 10^{3}=2.23 \times 10^{3}(\mathrm{~N})
\end{aligned}
$$

（d）This has already been noted in the previous part： $F_{b}=2.18 \times 10^{3} \mathrm{~N}$ ，and $T+F_{b}=m g$ ．
29．（a）Let $V$ be the volume of the block．Then，the submerged volume is $V_{s}=2 \mathrm{~V} / 3$ ．Since the block is floating，the weight of the displaced water is equal to the weight of the block，so $\rho_{w} V_{s}=\rho_{b} V$ ，where $\rho_{w}$ is the density of water，and $\rho_{b}$ is the density of the block．We substitute $V_{s}=2 \mathrm{~V} / 3$ to obtain

$$
\rho_{b}=2 \rho_{w} / 3=2\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right) / 3=6.7 \times 10^{2} \mathrm{~kg} / \mathrm{m}^{3} .
$$

（b）If $\rho_{o}$ is the density of the oil，then Archimedes＇ principle yields $\rho_{o} V_{s}=\rho_{b} V$ ．We substitute $V_{s}=$ 0.90 V to obtain $\rho_{o}=\rho_{b} / 0.90=7.4 \times 10^{2} \mathrm{~kg} / \mathrm{m}^{3}$ ．

43．Suppose that a mass $\Delta m$ of water is pumped in time $\Delta t$ ．The pump increases the potential energy of the water by $\Delta m g h$ ，where $h$ is the vertical distance through which it is lifted，and increases its kinetic energy by $(1 / 2) \Delta m v^{2}$ ，where $v$ is its final speed．The work it does is $W=\Delta m g h+(1 / 2) \Delta m v^{2}$ and its power is

$$
P=\frac{\Delta W}{\Delta t}=\frac{\Delta m}{\Delta t}\left(g h+\frac{1}{2} v^{2}\right)
$$

Now the rate of mass flow is $\Delta m / \Delta t=\rho_{w} A v$ ，where $\rho_{w}$ is the density of water and $A$ is the area of the hose．The area of the hose is $A=\pi r^{2}=\pi(0.010 \mathrm{~m})^{2}$ $=3.14 \times 10^{-4} \mathrm{~m}^{2}$ and $\rho_{w} A v=\left(1000 \mathrm{~kg} / \mathrm{m}^{3}\right)\left(3.14 \times 10^{-4}\right.$ $\left.\mathrm{m}^{2}\right)(5.00 \mathrm{~m} / \mathrm{s})=1.57 \mathrm{~kg} / \mathrm{s}$ ．Thus， $P=\rho A v\left[\rho g+\frac{1}{2} v^{2}\right]=(1.57 \mathrm{~kg} / \mathrm{s})\left[\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~m})\right.$

$$
\left.+(1 / 2)(5.0 \mathrm{~m} / \mathrm{s})^{2}\right]=66 \mathrm{~W}
$$

45．（a）We use the Eq．of continuity：$A_{1} v_{1}=A_{2} v_{2}$ ． Here $A_{1}$ is the area of the pipe at the top and $v_{1}$ is the speed of the water there；$A_{2}$ is the area of the
pipe at the bottom and $v_{2}$ is the speed of the water there．Thus $\quad v_{2}=\left(A_{1} / A_{2}\right) v_{1}=\left[\left(4.0 \mathrm{~cm}^{2}\right) /\left(8.0 \mathrm{~cm}^{2}\right)\right]$ $(5.0 \mathrm{~m} / \mathrm{s})=2.5 \mathrm{~m} / \mathrm{s}$ ．
（b）We use the Bernoulli Eq．：$p_{1}+(1 / 2) \rho v_{1}{ }^{2}+\rho g h_{1}=p_{2}$ $+(1 / 2) \rho v_{2}^{2}+\rho g h_{2}$ ，where $\rho$ is the density of water， $h_{1}$ is its initial altitude，and $h_{2}$ is its final altitude． Thus

$$
\begin{array}{r}
p_{2}=p_{1}+\frac{1}{2} \rho\left(v_{1}{ }^{2}-v_{2}^{2}\right)+\rho g\left(h_{1}-h_{2}\right) \\
=1.5 \times 10^{5}+\frac{1}{2}\left(1.0 \times 10^{3}\right)\left(5.0^{2}-2.5^{2}\right) \\
+\left(1.0 \times 10^{3}\right)(9.8)(10)=2.6 \times 10^{5}(\mathrm{~Pa})
\end{array}
$$

54．（a）The volume of water（during 10 minutes）is $V=\left(v_{1} t\right) A_{1}=(15 \mathrm{~m} / \mathrm{s})(10 \mathrm{~min})(60 \mathrm{~s} / \mathrm{min})(\pi / 4)(0.03$ $\mathrm{m})^{2}=6.4 \mathrm{~m}^{3}$ ．（b）The speed in the left section of pipe is

$$
v_{2}=v_{1}\left(\frac{A_{1}}{A_{2}}\right)=v_{1}\left(\frac{d_{1}}{d_{2}}\right)^{2}=(15)\left(\frac{3.0}{5.0}\right)^{2}=5.4(\mathrm{~m} / \mathrm{s})
$$

（c）Since $p_{1}+(1 / 2) \rho v_{1}^{2}+\rho g h_{1}=p_{2}+(1 / 2) \rho v_{2}^{2}+\rho g h_{2}$ and $h_{1}=h_{2}, p_{1}=p_{0}$ ，which is the atmospheric pressure，
$p_{2}=p_{0}+\frac{1}{2} \rho\left(v_{1}{ }^{2}-v_{2}^{2}\right)=1.01 \times 10^{5}+\frac{1}{2}\left(1.0 \times 10^{3}\right)$

$$
\left(15^{2}-5.4^{2}\right)=1.99 \times 10^{5}(\mathrm{~Pa})=1.97(\mathrm{~atm})
$$

Thus the gauge pressure is $(1.97 \mathrm{~atm}-1.00 \mathrm{~atm})=$ $0.97 \mathrm{~atm}=9.8 \times 10^{4} \mathrm{~Pa}$ ．
55．（a）Since S．P．14－8 deals with a similar situation， we use the final Eq．（labeled＂Answer＂）from it：$v$ $=(2 g h)^{1 / 2} \Rightarrow v=v_{0}$ for the projectile motion．The stream of water emerges horizontally（ $\theta_{0}=0^{\circ}$ in the notation of Chapt．4），and setting $y-y_{0}=-(H-h)$ in Eq．4－22，we obtain the＂time－of－flight＂

$$
t=\sqrt{\frac{-2(H-h)}{-g}}=\sqrt{\frac{2}{g}(H-h)} .
$$

Using this in Eq．4－21，where $x_{0}=0$ by choice of coordinate origin，we find

$$
\begin{aligned}
x=v_{0} t= & \sqrt{2 g h} \sqrt{2(H-h) / g}=2 \sqrt{h(H-h)} \\
& =2 \sqrt{10(40-10)}=35(\mathrm{~m}) .
\end{aligned}
$$

（b）The result of part（a）（which，when squared， reads $x^{2}=4 h(H-h)$ ）is a quadratic Eq．for $h$ once $x$ and $H$ are specified．Two solutions for $h$ are there－ fore mathematically possible，but are they both physically possible？For instance，are both solutions positive and less than $H$ ？We employ the quadratic formula：

$$
h^{2}-H h+\frac{1}{4} x^{2}=0 \Rightarrow h=\frac{1}{2}\left(H \pm \sqrt{H^{2}-x^{2}}\right),
$$

which permits us to see that both roots are physi－ cally possible，so long as $x \leq H$ ．Labeling the larger root $h_{1}$（where the plus sign is chosen）and the smaller root as $h_{2}$（where the minus sign is chosen）， then we note that their sum is simply

$$
h_{1}+h_{2}=\frac{1}{2}\left(H+\sqrt{H^{2}-x^{2}}\right)+\frac{1}{2}\left(H-\sqrt{H^{2}-x^{2}}\right)=H .
$$

Thus，one root is related to the other（generically
labeled $h^{\prime}$ and $h$ ）by $h^{\prime}=H-h$ ．Its numerical value is $h^{\prime}=40 \mathrm{~cm}-10 \mathrm{~cm}=30 \mathrm{~cm}$ ．（c）We wish to maximize the function $f=x^{2}=4 h(H-h)$ ．We differ－ entiate with respect to $h$ and set equal to zero to obtain

$$
d f / d h=4 H-8 h=0 \quad \Rightarrow \quad h=H / 2,
$$

or $h=(40 \mathrm{~cm}) / 2=20 \mathrm{~cm}$ ，as the depth from which an emerging stream of water will travel the maxi－ mum horizontal distance．
69．（a）We consider a point $D$ on the surface of the liquid in the container，in the same tube of flow with points $A, B$ and $C$ ．Applying Bernoulli＇s Eq．to points $D$ and $C$ ，we obtain

$$
p_{D}+\frac{1}{2} \rho v_{D}^{2}+\rho g h_{D}=p_{C}+\frac{1}{2} \rho v_{C}^{2}+\rho g h_{C},
$$

which leads to

$$
v_{C}=\sqrt{\frac{2\left(p_{D}-p_{C}\right)}{\rho}+2 g\left(h_{D}-h_{C}\right)+v_{D}^{2}} \approx \sqrt{2 g\left(d+h_{2}\right)} .
$$

where in the last step we set $p_{D}=p_{C}=p_{\text {air }}$ and $v_{D} / v_{C}$ $\approx 0$ ．Plugging in the values，we obtain

$$
v_{c}=\sqrt{2(9.8)(0.40+0.12)}=3.2(\mathrm{~m} / \mathrm{s})
$$

（b）We now consider points $B$ and $C$ ：

$$
p_{B}+\frac{1}{2} \rho v_{B}^{2}+\rho g h_{B}=p_{C}+\frac{1}{2} \rho v_{C}^{2}+\rho g h_{C} .
$$

Since $v_{B}=v_{C}$ by Eq．of continuity，and $p_{C}=p_{\text {air }}$ ， Bernoulli＇s Eq．becomes

$$
\begin{aligned}
& p_{B}=p_{C}+\rho g\left(h_{C}-h_{B}\right)=p_{\text {air }}-\rho g\left(h_{1}+h_{2}+d\right) \\
& =1.0 \times 10^{5}-\left(1.0 \times 10^{3}\right)(9.8)(0.25+0.40+0.12) \\
& =9.2 \times 10^{4}(\mathrm{~Pa})
\end{aligned}
$$

（c）Since $p_{B} \geq 0$ ，we must let $p_{\text {air }}-\rho g\left(h_{1}+d+h_{2}\right) \geq$ 0 ，which yields

$$
h_{1} \leq h_{1, \max }=\frac{p_{\text {air }}}{\rho}-d-h_{1} \leq \frac{p_{\text {air }}}{\rho}=10.3 \mathrm{~m} .
$$

80．${ }^{\text {• }}$ The absolute pressure is

$$
\begin{aligned}
& p=p_{0}+\rho g h=1.01 \times 10^{3} \mathrm{~Pa}+\left(1.03 \times 10^{3} \mathrm{~kg} / \mathrm{m}^{3}\right) \\
&\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(150 \mathrm{~m})=1.62 \times 10^{4} \mathrm{~Pa}
\end{aligned}
$$

 $10^{7}$ Pa．（b）By definition，$p=p_{g}+p_{\text {atm }}=1.22 \times 10^{7}$ Pa．（c）We interpret the question as asking for the total force compressing the sphere＇s surface，and we multiply the pressure by total area：$p\left(4 \pi r^{2}\right)=3.82$ $\times 10^{5} \mathrm{~N}$ ．（d）The（upward）buoyant force exerted on the sphere by the seawater is $F_{b}=\rho_{w} g V$ ，where $V=$ $(4 / 3) \pi r^{3}$ ．Therefore，$F_{b}=5.26 \mathrm{~N}$ ．（e）Newton＇s second law applied to the sphere（of mass $m=7.00 \mathrm{~kg}$ ） yields $F_{b}-m g=m a$ ，which results in $a=-9.04$ $\mathrm{m} / \mathrm{s}^{2}$ ，which means the acceleration vector has a magnitude of $9.04 \mathrm{~m} / \mathrm{s}^{2}$ ．（f）The direction is down－ ward．
92．＂（a）We assume that the top surface of the slab is at the surface of the water and that the automobile is at the center of the ice surface．Let $M$ be the mass of the automobile，$\rho_{i}$ be the density of ice，and $\rho_{w}$ be the density of water．Suppose the ice slab has
area $A$ and thickness $h$ ．Since the volume of ice is $A h$ ，the downward force of gravity on the auto－ mobile and ice is $\left(M+\rho_{\mathrm{i}} A h\right) g$ ．The buoyant force of the water is $\rho_{w} A h g$ ，so the condition of equilibrium is $\left(M+\rho_{i} A h\right) g-\rho_{w} A h g=0$ and

$$
A=\frac{M}{\left(\rho_{w}-\rho_{i}\right) h}=\frac{1100}{(998-917)(0.30)}=45\left(\mathrm{~m}^{2}\right)
$$

These density values are found in Table 14－1 of the text． （b）It does matter where the car is placed since the ice tilts if the automobile is not at the center of its surface．
91．Equilibrium of forces（on the floating body）is expressed as $F_{b}=m_{\text {body }} g \Rightarrow \rho_{\text {liquid }} g V_{\text {submerged }}=$ $\rho_{\text {body }} g V_{\text {total }}$ ，which leads to

$$
\frac{V_{\text {submerged }}}{V_{\text {total }}}=\frac{\rho_{\text {body }}}{\rho_{\text {liquid }}} .
$$

We are told（indirectly）that two－thirds of the body is below the surface，so the fraction above is $2 / 3$ ．Thus， with $\rho_{\text {body }}=0.98 \mathrm{~g} / \mathrm{cm}^{3}$ ，we find $\rho_{\text {liquid }} \approx 1.5 \mathrm{~g} / \mathrm{cm}^{3}$－ certainly much more dense than normal seawater （the Dead Sea is about seven times saltier than the ocean due to the high evaporation rate and low rainfall in that region）．
59．＊（a）The continuity Eq．yields $A v=a V$ ，and Bernoulli＇s Eq．yields $\Delta p+(1 / 2) \rho v^{2}=(1 / 2) \rho V^{2}$ ，where $\Delta p=p_{1}-p_{2}$ ．The first Eq．gives $V=(A / a) v$ ．We use this to substitute for $V$ in the second Eq．，and obtain $\Delta p+(1 / 2) \rho v^{2}=(1 / 2) \rho(A / a)^{2} V^{2}$ ．We solve for $v$ ．The result is

$$
v=\sqrt{\frac{2 \Delta p}{\rho\left[\left(A^{2} / a^{2}\right)-1\right]}}=\sqrt{\frac{2 a^{2} \Delta p}{\rho\left(A^{2}-1\right)}} .
$$

（b）We substitute values to obtain

$$
v=\sqrt{\frac{2\left(32 \times 10^{-4}\right)\left(55 \times 10^{3}-41 \times 10^{3}\right)}{(1000)\left[\left(64 \times 10^{-4}\right)^{2}-\left(32 \times 10^{-4}\right)^{2}\right]}}=3.06(\mathrm{~m} / \mathrm{s})
$$

Consequently，the flow rate is $A v=\left(64 \times 10^{-4} \mathrm{~m}^{2}\right)$ $(3.06 \mathrm{~m} / \mathrm{s})=2.0 \times 10^{-2} \mathrm{~m}^{3} / \mathrm{s}$ ．
61．＊（a）Bernoulli＇s Eq．gives $p_{A}=p_{B}+(1 / 2) \rho_{\text {air }} v^{2}$ ，But $\Delta p=p_{A}-p_{B}=\rho g h$ in order to balance the pressure in the two arms of the U－tube．Thus $\rho g h=(1 / 2) \rho_{\text {air }} \nu^{2}$ ，
or

$$
v=\sqrt{\frac{2 \rho g h}{\rho_{\text {air }}}}
$$

（b）The plane＇s speed relative to the air is

$$
v=\sqrt{\frac{2(810)(9.8)(0.260)}{1.03}}=63.3(\mathrm{~m} / \mathrm{s})
$$

－靜流體壓力 $(v=0)$ ：壓力 $p+$ 重力位能密度 $\rho g y$ $=$ 常數 連續方程式：體流率 $A v=$ 常數 柏努利方程式：壓力 $p+$ 動能密度 $(1 / 2) \rho v^{2}+$ 重力位能密度 $\rho g y=$ 常數 亞基米得原理：一部份或完全浸於流體內的物體，被一等於其排開流體重量之力所浮升，即浮力 $F_{B}=\rho_{f} V g$ 。

## 重點整理—第14章 流體

什麼造成抓地效應？又為什麼它會消失？密度 任一物質的密度 $\rho$ 定義爲每單位體積的質量：

$$
\begin{equation*}
\rho=\Delta m / \Delta V . \tag{14-1}
\end{equation*}
$$

通常物質試樣遠大於原子尺度，14－1 式可寫爲

$$
\begin{equation*}
\rho=m / V \tag{14-2}
\end{equation*}
$$

流體壓力 流體爲可流動的物質；其因無法承受切應力，形狀隨容器形狀而改變；然而它能施加一垂直於其表面的力，該力可藉壓力 $p$ 以描述：

$$
\begin{equation*}
p=\Delta F / \Delta A \tag{14-3}
\end{equation*}
$$

其中 $\Delta F$ 爲作用於面積 $\Delta A$ 之表面元素的力。假如力均匀施於平坦表面上，則 14－3 式可寫爲

$$
\begin{equation*}
p=F / A \tag{14-4}
\end{equation*}
$$

於流體中某特定點由流體壓力產生的力，其大小於各方向均相等。計示壓力爲在某點真實的壓力 （或絕對壓力）與大氣壓力之差。
壓力隨高度及深度變化 在静止流體內壓力隨鉛直位置 $y$ 而改變，對 $y$ 向上取正而言，

$$
\begin{equation*}
p_{2}=p_{1}+\rho g\left(y_{1}-y_{2}\right) \tag{14-7}
\end{equation*}
$$

流體內所有同一高度的點之壓力均相等。假如 $h$爲流體試樣在壓力爲 $p_{0}$ 某參考高度之下的深度，14－7 式變爲

$$
\begin{equation*}
p=p_{0}+\rho g h \tag{14-8}
\end{equation*}
$$

式中 $p$ 爲試樣內的壓力。

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J2．百年來人類飛行動力的演進，蔣小偉，科學發展 378期（9306） 42 。
J3．大氣中的狂暴舞者—龍捲風，李健成，科學發展 378期（9306） 52 。
J4．旋，還是不旋一變化球問題，李志揚，陳政宏，科學發展 388 期（9404） 54 。
J7．飛行夢想的實踐者—萊特兄弟，王裕宏，科學發展 376 期（9304）6。
J5．Q\＆A．高爾夫球上的小洞有什麼作用？科學人 2004年5月號。
J6．Q\＆A．正常血壓值為什麼應小於 120／80？為什麼不會隨身高改變？科學人2005年5月。
J8．耀眼的六百年一鄭和與〈鄭和航海圖〉，龍村倪，科學發展 379 期（9307）42。

帕斯卡原理 施於密閉容器中流體之壓力改變量，必不滅地傳遞至流體母一部份與容器壁上。亞基米德原理 當物體完全或部份浸於流體中，週遭的流體作用於物體造成一浮力 $\vec{F}_{b}$ 。此力垂直向上且其大小爲 $\quad F_{b}=m_{f} g$ ，
式中 $m_{f}$ 爲物體所排開流體的質量（亦即被物體排擠出的流體）。

當物體浮於流體上，作用於物體的（向上）浮力大小 $F_{b}$ 等於作用於物體的（向下）重力大小 $F_{g} \circ$ 浮力作用的物體之視重與真實重量的關係爲

$$
\begin{equation*}
\text { weight }_{a p p}=\text { weight }-F_{b} \tag{14-19}
\end{equation*}
$$

理想流體流動 理想流體爲不可壓縮且無黏滯性，並且其流動爲穩定的且無旋的。流線爲各流體質點所依循的路徑。流動管爲一整束流線。於任一流動管中流動遵循連續方程式：

$$
\begin{equation*}
R_{v}=A v=\text { 常數, } \tag{12-25}
\end{equation*}
$$

其中 $R_{v}$ 爲體積流率，$A$ 爲流管中任意點的截面積，而 $v$ 爲該點流體速率。質量流率 $R_{m}$ 爲

$$
\begin{equation*}
R_{m}=\rho R_{v}=\rho A v=\text { 常數. } \tag{14-25}
\end{equation*}
$$

柏努利方程式 應用力學能守恆原理於理想流體之流動導出沿著任意流動管之柏努利方程式：

$$
\begin{equation*}
p+\frac{1}{2} \rho v^{2}+\rho g y=\text { 常數 } \tag{14-29}
\end{equation*}
$$

（如發現錯誤煩請告知 jyang＠mail．ntou．edu．tw，Thanks．）
Archimedes 亞基米得；Bernoulli 柏努利；Pascal 帕斯卡； Rayleigh 瑞立；Torricelli 托里切利；fluid 流體；flow 流動；tube of flow 流動管；laminar flow；層流；equation of continuity 連續方程式；mass／volume flow rate 質／體流率；Mercury barometer 水銀氣壓計；open－tube manome－ ter 開管壓力計；pressure 壓力；absolute／gauge pressure絕對／計示壓力；systolic／diastolic pressure 收縮／舒張壓； mm Hg 毫米永柱；dam 水壩；hydraulic lever 液壓䫅桿； flap 副翼；siphon 虹吸管；wing 尾翼，擾流板；buoyant force 浮力；density 密度；desperado 歹徒；dye 染料； Ethanol 乙醇；Heimlich maneuver 海姆利胥急救法； negative lift 負升力；nonviscous 無黏滞的；pilot tube 領示管；irrotational 無旋的；scuba 水肺；streamline 流線； torpedo 魚雷；turbulent 急，湍流的；venturi meter 文士里計量計；
－備忘錄•

