Chapter 13 Gravitation

03. The gravitational force between the two parts is

$$F = Gm(M-m)/r^2 = (G/r^2)(mM-m^2),$$

which we differentiate with respect to m and set equal to zero:

$$dF/dm = 0 = (G/r^2)(M-2m) \quad \Rightarrow \quad M = 2m.$$

which leads to the result $m/M = 1/2$.

04. Using $F = GmM/r^2$, we find that the topmost mass pulls upward on the one at the origin with 1.9×10^8 N, and the rightmost mass pulls rightward on the one at the origin with 1.0×10^8 N. Thus, the (x, y) components of the net force, which can be converted to polar components (here we use magnitude-angle notation), are

$$\vec{F}_{net} = (1.04\times10^{-8}, 1.85\times10^{-8}) \Rightarrow (2.13\times10^{-8}\angle 60.6°).$$

(**a**) The magnitude of the force is 2.13×10^8 N. (**b**) The direction of the force relative to the $+x$ axis is 60.6°.

11. If the lead sphere were not hollowed the magnitude of the force it exerts on m would be $F_1 = GmM/d^2$. Part of this force is due to material that is removed. We calculate the force exerted on m by a sphere that just fills the cavity, at the position of the cavity, and subtract it from the force of the solid sphere. The cavity has a radius $r = R/2$. The material that fills it has the same density (mass to volume ratio) as the solid sphere. That is $M_c/r^3 = M/R^3$, where M_c is the mass that fills the cavity. The common factor $4\pi/3$ has been canceled. Thus,

$$M_c = (r/R)\,M = (1/2)^3 M = (1/8)M\,.$$

The center of the cavity is $d - r = d - R/2$ from m, so the force it exerts on m is

$$F_2 = G\frac{(M/8)m}{d-R/2}.$$

With M (m) = 2.95 (0.431) kg and R (d) = 4.00 (9.00) cm, the force of the hollowed sphere on m is

$$F = F_1 - F_2 = GMm\left[\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right]$$

$$= \frac{GMm}{d^2}\left[1 - \frac{1}{8(1-R/2d)^2}\right] = 8.31\times10^{-9} \text{ (N)}.$$

09. (**a**) The distance between any of the spheres at the corners and the sphere at the center is

$$r = \ell/2\cos30° = \ell/\sqrt{3}\,,$$

where ℓ is the length of one side of the equilateral triangle. The net (downward) contribution caused by the two bottom-most spheres (each of mass m) to the total force on m_4 has magnitude

$$2F_y = 2\frac{Gm_4m}{r^2}\sin30° = 3\frac{Gm_4m}{\ell^2}.$$

This must equal the magnitude of the pull from M, so

$$3\frac{Gm_4m}{\ell^2} = \frac{Gm_4m}{(\ell/\sqrt{3})^2},$$

which readily yields $m = M$. (**b**) Since m_4 cancels in that last step, then the amount of mass in the center sphere is not relevant to the problem. The net force is still zero.

15. The acceleration due to gravity is given by $a_g = GM/r^2$, where M is the mass of Earth and r is the distance from Earth's center. We substitute $r = R+h$, where R is the radius of Earth and h is the altitude, to obtain $a_g = GM/(R+h)^2$. We solve for h and obtain $h = (GM/a_g)^{1/2} - R$. With $R = 6.37\times10^6$ m and $M = 5.98\times10^{24}$ kg, so

$$h = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{4.9}} - 6.37\times10^6 = 2.6\times10^6 \text{ (m)}.$$

20. (**a**) What contributes to the GmM/r^2 force on m is the (spherically distributed) mass M contained within r (where r is measured from the center of M). At point A we see that M_1+M_2 is at a smaller radius than $r = a$ and thus contributes to the force:

$$|F_{on,m}| = G\frac{(M_1+M_2)}{a^2}m.$$

(**b**) In the case $r = b$, only M_1 is contained within that radius, so the force on m becomes GM_1m/b^2. (**c**) If the particle is at C, then no other mass is at smaller radius and the gravitational force on it is zero.

26. The gravitational potential energy is

$$U = -Gm(M-m)/r = -(G/r)(mM-m^2),$$

which we differentiate with respect to m and set equal to zero (in order to minimize). Thus, we find $M-2m = 0$ which leads to the ratio $m/M = 1/2$ to obtain the least potential energy. Note that a second derivative of U with respect to m would lead to a positive result regardless of the value of m which means its graph is everywhere concave upward and thus its extremum is indeed a minimum.

33. (**a**) We use the principle of conservation of energy. Initially the particle is at the surface of the asteroid and has potential energy $U_i = GMm/R$, where M is the mass of the asteroid, R is its radius, and m is the mass of the particle being fired upward. The initial kinetic energy is $(½)mv^2$. The particle just escapes if its kinetic energy is zero when it is infinitely far from the asteroid. The final potential and kinetic energies are both zero. Conservation of energy yields $GMm/R + (½)mv^2 = 0$. We replace GM/R with a_gR, where a_g is the acceleration due to gravity at the surface. Then, the energy eq. becomes $a_gR + (½)v^2 = 0$. We solve for v:

$$v = \sqrt{2a_gR} = \sqrt{2(3.0)(500\times10^3} = 1.7\times10^3 \text{ (m/s)}.$$

(**b**) Initially the particle is at the surface; the potential energy is $U_i = GMm/R$ and the kinetic energy is $K_i = (½)mv^2$. Suppose the particle is a distance h

above the surface when it momentarily comes to rest. The final potential energy is $U_f = GMm/(R+h)$ and the final kinetic energy is $K_f = 0$. Conservation of energy yields

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=-\frac{GMm}{R+h}.$$

We replace GM with a_gR^2 and cancel m in the energy eq. to obtain

$$-a_gR+\frac{1}{2}v^2=-a_g\frac{R^2}{R+h}.$$

The solution for h is

$$h=\frac{2a_gR^2}{2a_gR^2-v^2}-R=\frac{2(3.0)(500\times10^3)}{2(3.0)(500\times10^3)-1000}$$
$$-500\times10^3=2.5\times10^5\text{ (m)}.$$

(**c**) Initially the particle is a distance h above the surface and is at rest. Its potential energy is $U_i = GMm/(R+h)$ and its initial kinetic energy is $K_i = 0$. Just before it hits the asteroid its potential energy is $U_f = GMm/R$. Write $(½)mv_f^2$ for the final kinetic energy. Conservation of energy yields

$$-\frac{GMm}{R+h}=-\frac{GMm}{R}+\frac{1}{2}mv^2.$$

We substitute a_gR^2 for GM and cancel m, obtaining

$$-a_g\frac{R^2}{R+h}=-a_gR+\frac{1}{2}v^2.$$

The solution for v is

$$v=\sqrt{2a_gR-\frac{2a_gR^2}{R+h}}=1.4\times10^3\text{ (m/s)}$$
$$=\sqrt{2(3.0)(500\times10^3)-\frac{2(3.0)(500\times10^3)}{500\times10^3+1000\times10^3}}.$$

37. Let $m = 0.020$ kg and $d = 0.600$ m (the original edge-length, in terms of which the final edge-length is $d/3$). The total initial gravitational potential energy (using Eq. 13-21 and some elementary trigonometry) is

$$U_i=-\frac{4Gm^2}{d}-\frac{2Gm^2}{\sqrt{2}d}.$$

Since U is inversely proportional to r then reducing the size by 1/3 means increasing the magnitude of the potential energy by a factor of 3, so

$$U_f=3U_i\Rightarrow U=2U_i=2\,(4+\sqrt{2})(-\frac{Gm^2}{d})$$
$$=-4.82\times10^{-13}\text{ (J)}.$$

43. (**a**) If r is the radius of the orbit then the magnitude of the gravitational force acting on the satellite is given by GMm/r^2, where M is the mass of Earth and m is the mass of the satellite. The magnitude of the acceleration of the satellite is given by v^2/r, where v is its speed. Newton's second law yields $GMm/r^2 = mv^2/r$. Since the radius of Earth is 6.37×10^6 m the orbit radius is $r = 6.37\times10^6$ m $+160\times10^3$ m $= 6.53\times10^6$ m. The solution for v is

$$v=\sqrt{\frac{GM}{r}}=\sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{6.53\times10^6}}$$
$$=7.82\times10^3\text{ (m/s)}.$$

(**b**) Since the circumference of the circular orbit is $2\pi r$, the period is

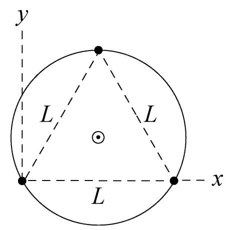
$$T=\frac{2\pi r}{v}=\frac{2\pi(6.53\times10^6\text{ m})}{6.53\times10^3\text{ m/s}}=5.25\times10^3\text{ s}=87.5\text{ min}.$$

46. To "hover" above Earth ($M_E = 5.98\times10^{24}$ kg) means that it has a period of 24 hours (86400 s). By Kepler's law of periods,

$$(86400)^2=\frac{4\pi^2}{GM_E}r^3\Rightarrow r=4.225\times10^7\text{ m}.$$

Its altitude is therefore $r - R_E$ (where $R_E = 6.37\times10^6$ m) which yields 3.58×10^7 m.

53. Each star is attracted toward each of the other two by a force of magnitude GM^2/L^2, along the line that joins the stars. The net force on each star has magnitude $2(GM^2/L^2)\cos30°$ and is directed toward the center of the triangle. This is a centripetal force and keeps the stars on the same circular orbit if their speeds are appropriate. If R is the radius of the orbit, Newton's second law yields $(GM^2/L^2)\cos30° = Mv^2/R$. The stars rotate about their center of mass (marked by a circled dot on the diagram above) at the intersection of the perpendicular bisectors of the triangle sides, and the radius of the orbit is the distance from a star to the center of mass of the three-star system. We take the coordinate system to be as shown in the diagram, with its origin at the left-most star. The altitude of an equilateral triangle is $\sqrt{3}\,L/2$, so the stars are located at $x = 0, y = 0$; $x = L, y = 0$; and $x = L/2, y = \sqrt{3}\,L/2$. The x coordinate of the center of mass is $x_c = (L+L/2)/3 = L/2$ and the y coordinate is $y_c = (\sqrt{3}/2)L/3 = L/2\sqrt{3}$. The distance from a star to the center of mass is



$$R=\sqrt{x_c^2+y_c^2}=\sqrt{(L^2/4)+(L^2/12)^2}=L/\sqrt{3}.$$

Once the substitution for R is made Newton's second law becomes $(2GM^2/L^3)\cos30° = \sqrt{3}\,Mv^2/L$. This can be simplified somewhat by recognizing that $\cos30° = \sqrt{3}/2$, and we divide the eq. by M. Then, $GM/L^2 = v^2/L$ and $v = (GM/L)^{1/2}$.

57. The energy required to raise a satellite of mass m to an altitude h (at rest) is given by

$$E_1=\Delta U=GM_Em(\frac{1}{R_E}-\frac{1}{R_E+h}),$$

and the energy required to put it in circular orbit once it is there is

$$E_2 = \frac{1}{2}mv_{\text{orb}}^2 = GM_E m\frac{1}{2(R_E + h)}.$$

Consequently, the energy difference is

$$\Delta E = E_1 - E_2 = GM_E m[\frac{1}{R_E} - \frac{3}{2(R_E + h)}].$$

(**a**) Solving the above eq., the height h_0 at which $\Delta E = 0$ is given by

$$\frac{1}{R_E} - \frac{3}{2(R_E + h_0)} = 0 \Rightarrow h_0 = \frac{R_E}{2} = 3.19\times10^6 \text{ (m)}.$$

(**b**) For greater height $h > h_0$, $\Delta E > 0$ implying $E_1 > E_2$. Thus, the energy of lifting is greater.

71. (**a**) With $M = 2.0\times10^{30}$ kg and $r = 10^4$ m, we find

$$a_g = \frac{GM}{r^2} = 1.3\times10^{12} \text{ (m/s}^2).$$

(**b**) Although a close answer may be obtained by using the constant acceleration eqs. of Chapter 2, we show the more general approach (using energy conservation):

$$K_0 + U_0 = K + U,$$

where $K_0 = 0$, $K = (½)mv^2$ and U given by Eq. 13-21. Thus, with $r_o = 10001$ m, we find

$$v = \sqrt{2GM(\frac{1}{r} - \frac{1}{r_0})} = 1.6\times10^6 \text{ (m/s)}.$$

76. (**a**) Since the volume of a sphere is $4\pi R^3/3$, the density is

$$\rho = M_{\text{total}}/(4\pi R^3/3) = (3M_{\text{total}})/(4\pi R^3).$$

When we test for gravitational acceleration (caused by the sphere, or by parts of it) at radius r (measured from the center of the sphere), the mass M which is at radius less than r is what contributes to the reading (GM/r^2). Since $M = \rho(4\pi r^3/3)$ for $r < R$ then we can write this result as

$$G\frac{(3M_{\text{total}}/4\pi R^3)(4\pi r^3/3)}{r^2} = G\frac{M_{\text{total}}r}{R^3},$$

when we are considering points on or inside the sphere. Thus, the value a_g referred to in the problem is the case where $r = R$:

$$a_g = \frac{GM_{\text{total}}}{R^2},$$

and we solve for the case where the acceleration equals $a_g/3$:

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}r}{R^3} \quad \Rightarrow \quad r = \frac{R}{3}.$$

(**b**) Now we treat the case of an external test point. For points with $r > R$ the acceleration is GM_{total}/r^2, so the requirement that it equal $a_g/3$ leads to

$$\frac{GM_{\text{total}}}{3R^2} = \frac{GM_{\text{total}}}{r^2} \quad \Rightarrow \quad r = \sqrt{3}\,R.$$

87. (**a**) Kepler's law of periods is

$$T^2 = \frac{4\pi^2}{GM}r^3.$$

With $M = 6.0\times10^{30}$ kg and $T = 300(86400) = 2.6\times10^7$ s, we obtain $r = 1.9\times10^{11}$ m. (**b**) The orbit is circular suggests that its speed is constant, so

$$v = 2\pi r/T = 4.6\times10^4 \text{ m/s}.$$

98. If the angular velocity were any greater, loose objects on the surface would not go around with the planet but would travel out into space. (**a**) The magnitude of the gravitational force exerted by the planet on an object of mass m at its surface is given by $F = GmM/R^2$, where M is the mass of the planet and R is its radius. According to Newton's second law this must equal mv^2/R, where v is the speed of the object. Thus,

$$GM/R^2 = v^2/R.$$

Replacing M with $(4\pi/3)\rho R^3$ (where ρ is the density of the planet) and v with $2\pi R/T$ (where T is the period of revolution), we find

$$(4\pi/3)G\rho R = (4\pi^2/T^2)R.$$

We solve for T and obtain $T = (3\pi/G\rho)^{1/2}$.

(**b**) With $\rho = 3.0\times10^3$ kg/m^3, we evaluate the eq. for T:

$$T = [3\pi/(6.67\times10^{-11})/(3.0\times10^3)]^{1/2} = 6.86\times10^3 \text{ (s)} = 1.9 \text{ (h)}.$$

99. Let v and V be the speeds of particles m and M, respectively. These are measured in the frame of reference described in the problem (where the particles are seen as initially at rest). Now, momentum conservation demands

$$mv = MV \quad \Rightarrow \quad v + V = v(1 + \frac{m}{M}),$$

where $v+V$ is their relative speed (the instantaneous rate at which the gap between them is shrinking). Energy conservation applied to the two-particle system leads to

$$K_i + U_i = K + U,$$

$$0 - \frac{GmM}{r} = \frac{1}{2}mv^2 + \frac{1}{2}MV^2 - \frac{GmM}{d}.$$

$$-\frac{GmM}{r} = \frac{1}{2}mv^2(1 + \frac{m}{M}) - \frac{GmM}{d}.$$

If we take the initial separation r to be large enough that GmM/r is approximately zero, then this yields a solution for the speed of particle m:

$$v = \sqrt{\frac{2GM}{d(1 + m/M)}}.$$

Therefore, the relative speed is

$$v + V = \sqrt{\frac{2GM}{d(1 + m/M)}}\,(1 + \frac{m}{M}) = \sqrt{\frac{2G(M + m)}{d}}.$$

103. The magnitude of the net gravitational force on one of the smaller stars (of mass m) is

$$\frac{GMm}{r^2} + \frac{Gmm}{(2r)^2} = \frac{Gm}{r^2}(M + \frac{m}{4}).$$

This supplies the centripetal force needed for the motion of the star:

$$\frac{Gm}{r^2}(M+\frac{m}{4})=m\frac{v^2}{r}, \quad \text{where } v=\frac{2\pi r}{T}.$$

Plugging in for speed v, we arrive at an eq. for period T:

$$T=\frac{2\pi r^{3/2}}{\sqrt{G(M+m/4)}}.$$

94.* (**a**) We partition the full range into arcs of 3° each: 360°/3° = 120. Thus, the maximum number of geosynchronous satellites is 120. (**b**) Kepler's law of periods, applied to a satellite around Earth, gives $T^2 = r^3(4\pi^2/GM_s)$, where T = 24 h = 86400 s for the geosynchronous case. Thus, we obtain $r = 4.23\times10^7$ m. (**c**) The arc length s is related to angle of arc θ (in radians) by $s = r\theta$. Thus, with $\theta = 3(\pi/180) = 0.052$ rad, we find $s = 2.2\times10^6$ m. (**d**) Points on the surface (which, of course, is not in orbit) are moving toward the east with a period of 24 h. If the satellite is found to be east of its expected position (above some point on the surface for which it used to stay directly overhead), then its period must now be *smaller* than 24 h. (**e**) From Kepler's law of periods, it is evident that smaller T requires smaller r. The storm moved the satellite towards Earth.

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

第 13 章 重力作用

位於宇宙中心的怪物為何？

人外有人，星外有星；白馬非馬，黑洞非洞

$$\vec{F}_{12} = -G(m_1m_2/r_{12}^2)\,\hat{r}_{21} = -\vec{F}_{21};$$

重力定律 牛頓在分析行星繞太陽運動時，發現 "重力定律" (1687 年發表) "**在宇宙中任意質點間以一引力相互吸引，而此力與質點質量乘積成正比，與它們之間的距離之平方成反比**"。

$F_g = Gm_1m_2/r_{12}^2$，$G = 6.672\times10^{-11}$ N·m^2·kg^{-2}

①重力係沿著兩質點之連線方向；②重力形成一作用力及反作用力對；③重力遵守疊加原理 $\vec{F}_1 = \vec{F}_{12}+\vec{F}_{13}+\vec{F}_{14}+\ldots$；④重力與物體之運動狀態無關。$G$ 值: 1798，Cavendish 首次實驗測出。

重力之殼層定理：***A.***對一均勻球(殼)而言，其與球(殼)外心質點之動作用儼然整個球(殼)之質量集中於球(殼)心。***B.***對一均勻殼而言，其與殼內質點之重力作用為零。***C.***對一均勻球而言，其與球內質點之重力作用與到球心距離成正比。

<**証明**>：可視球為半徑為 r 之實心球(a)及剩餘之外殼(b)組成，而(b)之貢獻為零，因此

$$F = GM_am/r^2 = GM(r/R)^3m/r^2 = G(Mm/R^3)\,r.$$

重力位能：重力為保守力 ⇒ 重力位能

$$U(\vec{r}_A)-U(\vec{r}_B)\equiv-\int_{\vec{r}_B}^{\vec{r}_A}\vec{F}_{12}\cdot d\vec{r}=\int_{r_B}^{r_A}\frac{Gm_1m_2}{r^2}dr$$

$U(r_A) - U(r_B) = -(Gm_1m_2/r_A)+(Gm_1m_2/r_B)$,

$U(r) = -Gm_1m_2/r$, setting $U(r) = 0$ as $r_B \to \infty$,

♦地表附近之重力位能：

$U(R+y) - U(R) = GmM_E/R_E - GmM_E/(R_E+y)$

$\cong GmM_Ey/R_E^2$, $U(y) = m(GM_E/R_E^2)y = mgy$.

重力位能 ＋ 動能 ＝ 常數

$$E = K + U = \tfrac{1}{2}mv^2 - GmM_E/r.$$

克普勒行星運動定律：**1.**第一(**軌道**)定律：太陽系之行星，各在以太陽為焦點之一橢圓軌道上運行。**2.**第二(**面積**)定律：由太陽連至行星之線，於相等時間中掃過相等的面積。**3.**第三(**週期**)定律：行星距太陽之平均距離 R 之立方，與行星繞太陽周期 T 之平方的比值 R^3/T^2，對各個行星皆相等。

♦衛星繞地球軌道如為圓形 $GmM/r^2 = mv^2/r$,

$v=\sqrt{GM/r}$, $K=\frac{1}{2}mv^2 = GmM/2r$,

$U = -GmM/r$, $T^2 = (4\pi^2/GM)r^3$, r = 軌道半徑.

♦衛星繞地球軌道如為橢圓形

$E = -GmM/2a$, $T^2 = (4\pi^2/GM)a^3$, a = 半長軸.

♦衛星高度：3.58×10^4 km (同步), 891 km (華二)

♦當人造衛星在大氣層運轉時，遭受空氣阻力，耗損力學能ΔE < 0，但動能(或速率)仍增加，此乃軌道半徑減小，而位能減少以致！

脫離速度：$v_{esc}\equiv\sqrt{2GM/R}$,

$E_R = E_\infty$, $\frac{1}{2}mv_R^2 - GmM/R = \frac{1}{2}mv_\infty^2 - GmM/r_\infty$,

Set $v_\infty = 0$, $r_\infty \to \infty$, $v_R=\sqrt{2GM/R}\equiv v_{esc}$.

** v_{esc} = 11.2 km/s (Earth), 4.3 km/s (Mercury)

J1.*Q&A.*在地球上貫穿出無底洞？科學人 2003 年 9 月號。 J2.*Q&A.*要怎麼量地球的體重呢？科學人 2004 年 7 月號。 J3.*Q&A.*要怎麼量地球的體重呢？科學人 2004 年 7 月號。 J4.*Q&A.*科學家怎麼知道地球內部的組成？科學人 2004 年 12 月號。 J5.*Q&A.*為什麼恆星會閃爍，行星不會？科學人 2005 年 12 月號。 J6.*Q&A.*太空船如何利用「彈弓效應」改變軌道？科學人 2006 年 1 月號。 J7.為什麼有質量？(物理學)，凱思，科學人 No. 42 (2005 年 8 月號) 56-63。 J8.宇宙的奧祕---令人驚艷的現代宇宙學，演講人吳俊輝（文字整理李健成），科學發展 374 期 (9302) 42。

•備忘錄•

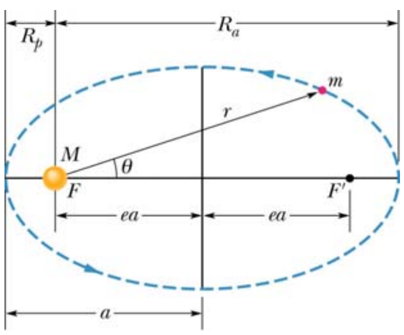
Table 1

	星球名稱	神話故事之象徵	衛星數目	軌道半徑	軌道週期	ε
0	太陽(Sun)			--	--	--
1	水星(Mercury)	使者及商業之神(羅)		5.80×10^{10}	88.0d	0.2056
2	金星(Venus)	司愛及美之女神(羅)		1.08×10^{11}	224.7d	0.0068
3	地球(Earth)		1	1.49×10^{11}	365.3d	0.0167
4	火星(Mars)	代表戰神(羅)	2	2.28×10^{11}	687.0d	0.0934
5	木星(Jupiter)	主神(羅)	61	7.78×10^{11}	11.86y	0.0485
6	土星(Saturn)	農神(羅)	31	1.43×10^{12}	29.46y	0.0555
7	天王星(Uranus)	天神(希)	25	2.87×10^{12}	84.02y	0.0463
8	海王星(Neptune)	海神(羅)	13	4.49×10^{12}	164.8y	0.0090
9	冥王星(Pluto)	冥府之神(希羅)	1	5.90×10^{12}	247.7y	0.2490

地球 5.98×10^{24} kg / 6.37×10^{6} m，月球 7.36×10^{22} kg，太陽 1.99×10^{30} kg

Table 2 Escape Velocities for the Planets, the Moon, and the Sun

Plants	Mercury	Venus	Earth	Moon	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto	Sun
v_{esc}(km/s)	4.3	10.3	11.2	2.38	5.0	59.5	36	22	24	1.1	618



F, *F'* 爲橢圓之焦點，設 *F* 爲太陽之位置

P(*A*)爲行星之近(遠)日點

直角座標(x, y)，極座標(r, θ)

$x^2/a^2 + y^2/b^2 = 1$，$a =$ 半長軸，$b =$ 半短軸

$\alpha/r = 1 - \varepsilon\cos\theta$，$\varepsilon \equiv$ eccentricity 離心率

$c = \underline{OF} = \varepsilon a,\ c^2 = a^2 - b^2,\ \varepsilon^2 = 1 - b^2/a^2.$