Chapter **10** Rotation

01. (**a**) The second hand of the smoothly running watch turns through 2π radians during 60 s. Thus, $\omega = 2\pi/60 = 0.105$ (rad/s). (**b**) The minute hand of the smoothly running watch turns through 2π radians during 3600 s. Thus, $\omega = 2\pi/3600 = 1.75\times10^{-3}$ (rad/s). (**c**) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus, $\omega = 2\pi/43200 = 1.45\times10^{-4}$ (rad/s).

02. The problem asks us to assume v_{cm} and ω are constant. For consistency of units, we write v_{cm} = (85 mi/h)(5280 ft/mi)/(60 min/h) = 7480 ft/min. Thus, with Δx = 60 ft, the time of flight is $t = \Delta x/v_{cm}$ = 60/7480 = 8.02×10^{-3} (min). During that time, the angular displacement of a point on the ball's surface is $\theta = \omega t$ = (1800 rev/min)(8.02×10^{-3} min) ≈ 14 rev.

04. If we make the units explicit, the function is

$$\theta = (4.0\text{ rad/s})\,t - (3.0\text{ rad/s}^2)\,t^2 + (1.0\text{ rad/s}^3)\,t^3,$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures. (**a**) Eq. 10-6 leads to

$$\omega = d\theta/dt$$
$$= (4.0\text{ rad/s}) - (6.0\text{ rad/s}^2)\,t + (3.0\text{ rad/s}^3)\,t^2.$$

Evaluating this at t = 2 s yields ω_2 = 4.0 rad/s. (**b**) Evaluating the expression in part (a) at t = 4 s gives ω_4 = 28 rad/s. (**c**) Consequently, Eq. 10-7 gives $\alpha_{av} = (\omega_4-\omega_2)/(t_4-t_2)$ = 12 rad/s^2. (**d**) And Eq. 10-8 gives

$$\alpha = d\omega/dt = -6.0\text{ rad/s}^2 + (6.0\text{ rad/s}^3)\,t\,.$$

Evaluating this at t = 2 s produces α_2 = 6.0 rad/s^2. (**e**) Evaluating the expression in part (d) at t = 4 s yields α_4 = 18 rad/s^2. We note that our answer for α_{av} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

10. We assume the sense of initial rotation is positive. Then, with ω_0 = +120 rad/s and ω = 0 (since it stops at time t), our angular acceleration ("deceleration") will be negative-valued: α = –4.0 rad/s^2. (**a**) We apply Eq. 10-12 to obtain t. $\omega = \omega_0 + \alpha t \Rightarrow t$ = (0–120)/(–4.0) = 30 (s). (**b**) And Eq. 10-15 gives

$$\theta = \omega_{av}t = \tfrac{1}{2}(\omega_0+\omega)t$$
$$= (120+0)(30)/2 = 1.8\times10^3\text{ (rad)}.$$

Alternatively, Eq.10-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular eq. in Table 10-1 (except Eq. 10-12) can be used to find θ.

15. The wheel has angular velocity ω_0 = +1.5 rad/s = +0.239 rev/s^2 at t = 0, and has constant value of angular acceleration α < 0, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at t = 0) is θ_1 = +20 rev, and at t_2 its angular displacement is θ_2 = +40 rev and its angular velocity is ω_2 = 0 . (**a**) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \tfrac{1}{2}(\omega_0+\omega_2)\,t_2 \quad\Rightarrow\quad t_2 = 2(40)/0.239,$$

which yields t_2 = 335 s which we round off to $t_2 \approx 3.4\times10^2$ s. (**b**) Any eq. in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \tfrac{1}{2}\,\omega_2\,t_2 - \tfrac{1}{2}\,\alpha\,t_2^2 \Rightarrow \alpha = -2(40)/335^2,$$

which yields $\alpha = -7.12\times10^{-4}$ rev/s^2 which we convert to α = -4.5×10^{-3} rad/s^2. (**c**) Using $\theta_1 = \omega_0 t_1 + (½)\alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{1}{\alpha}\left(-\omega_0 \pm \sqrt{\omega_0^2 + 2\alpha\theta_1}\right)$$
$$= \frac{-0.239 \pm \sqrt{0.239^2 + 2(-7.12\times10^{-4})(20)}}{-7.12\times10^{-4}}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is t_1 = 98 s.

21. (**a**) We obtain ω = (200 rev/min)(2π rad/rev)/(60 s/min) = 20.9 rad/s. (**b**) With r = 1.20/2 = 0.60 (m), Eq. 10-18 leads to $v = r\omega$ = (0.60)(20.9) = 12.5 (m/s). (**c**) With t = 1 min, ω = 1000 rev/min and ω_0 = 200 rev/min, Eq. 10-12 gives $\alpha = (\omega-\omega_0)/t$ = 800 rev/min^2. (**d**) With the same values used in part (c), Eq. 10-15 becomes $\theta = (½)(\omega_0+\omega)t$ = (200+1000)(1)/2 = 600 (rev).

26. (**a**) The tangential acceleration, using Eq. 10-22, is $a_t = \alpha r$ = (14.2 rad/s^2)(2.83 cm) = 40.2 cm/s^2 . (**b**) In rad/s, the angular velocity is ω = (2760)(2π/60) = 289 (rad/s), so $a_r = \omega^2 r$ = (289 rad/s)^2(2.83×10^{-2} m) = 2.36×10^3 m/s^2. (**c**) The angular displacement is, using Eq. 10-14, $\theta = \omega^2/2\alpha$ = 289^2/(2×14.2) = 2.94×10^3 (rad). Then, using Eq. 10-1, the distance traveled is $s = r\theta$ = (0.0283 m)(2.94×10^3 rad) = 83.2 m.

39. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(e) and the parallel-axis theorem (Eq. 10-36). (**a**) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$I = I_1 + I_2 + I_3 + I_4 = [\tfrac{1}{12}Md^2 + M(\tfrac{1}{2}d)^2] + md^2$$
$$+ [\tfrac{1}{12}Md^2 + M(\tfrac{3}{2}d)^2] + m(2d)^2 = \tfrac{8}{3}Md^2 + 5md^2$$
$$= \tfrac{8}{3}(1.2)(0.056)^2 + 5(0.85)(0.056)^2 = 0.023\ (\text{kg}\cdot\text{m}^2).$$

(**b**) Using Eq. 10-34, we have

$$K = \frac{1}{2} I\omega^2 = (\frac{4}{3} M + \frac{5}{2} m)d^2\omega^2$$
$$= [\frac{4}{3}(1.2)+\frac{5}{2}(0.85)](0.056)^2(0.30)^2 = 1.1\times10^{-3} \text{ (J)}.$$

41. We use the parallel-axis theorem. According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{cm} = \frac{1}{12} M(a^2 + b^2).$$

A parallel axis through the corner is a distance $h = [(a/2)^2+(b/2)^2]^{1/2}$ from the center. Therefore,

$$I = I_{cm}+Mh^2 = \frac{1}{12} M(a^2+b^2)+\frac{1}{4} M(a^2+b^2)=\frac{1}{3} M(a^2+b^2)$$
$$= (0.172)(0.035^2+0.084^2) = 4.7\times10^{-4} \text{ (kg}\cdot\text{m}^2).$$

47. We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1F_1\sin\theta_1$ is associated with $\boldsymbol{F}_1$ and a negative torque of magnitude $r_2F_2\sin\theta_2$ is associated with $\boldsymbol{F}_2$. The net torque is consequently $\tau = r_1F_1\sin\theta_1 - r_2F_2\sin\theta_2$. Substituting the given values, we obtain

$$\tau = (1.30 \text{ m})(4.20 \text{ N}) \sin75° - (2.15 \text{ m})(4.90 \text{ N}) \sin60° = -3.85 \text{ N}\cdot\text{m}.$$

51. Combining Eq. 10-45 ($\tau_{net} = I\alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega/t$ by Eq. 10-12 (with $\omega_0 = 0$). Using item (c) in Table 10-2 and solving for F_2, we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02(250)}{2\,(1.25)} + 0.1$$
$$= 0.140 \text{ (N)}.$$

55. (**a**) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block, then its coordinate is given by $y = (½)at^2$, so

$$a = 2y/t^2 = 2(0.750 \text{ m})/(5.00 \text{ s})^2 = 6.00\times10^{-2} \text{ m/s}^2.$$

The lighter block has an acceleration of 6.00×10^{-2} m/s^2 upward. (**b**) Newton's second law for the heavier block is $m_hg - T_h = m_ha$, where m_h is its mass and T_h is the tension force on the block. Thus,

$$T_h = m_h(g-a)= (0.500 \text{ kg})(9.8 \text{ m/s}^2 - 6.00\times10^{-2} \text{ m/s}^2) = 4.87 \text{ N}.$$

(**c**) Newton's second law for the lighter block is $m_\ell g - T_\ell = m_\ell a$, where T_ℓ is the tension force on the block. Thus, $T_\ell = m_\ell(g+a)$

$$= (0.460 \text{ kg})(9.8 \text{ m/s}^2 + 6.00\times10^{-2} \text{ m/s}^2) = 4.54 \text{ N}.$$

(**d**) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so $\alpha = a/R = (6.00\times10^{-2} \text{ m/s}^2)/(5.00\times10^{-2} \text{ m}) = 1.20 \text{ rad/s}^2$.

(**e**) The net torque acting on the pulley is $\tau = (T_h - T_\ell)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = (T_h-T_\ell)R/\alpha = (4.87 \text{ N} - 4.54 \text{ N})(5.00\times10^{-2} \text{ m})/(1.20 \text{ rad/s}^2) = 1.38\times10^{-2} \text{ kg}\cdot\text{m}^2.$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $(½)\ell$ from either end, its initial potential energy is $(½)mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $(½)I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2} mg\ell = \frac{1}{2} I\omega^2 \Rightarrow \omega = \sqrt{mg\ell / I}\ .$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \ell\omega = \sqrt{mg\ell^3 / I}\ .$$

Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = (⅓)m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3(9.80 \text{ m/s}^2)(1.00 \text{ m})} = 5.42 \text{ m/s}.$$

67. From Table 10-2, the rotational inertia of the spherical shell is $(⅔)MR^2$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}(\frac{2}{3} MR^2)\,\omega_{sphere}^2 + \frac{1}{2} I\omega_{pulley}^2 + \frac{1}{2} mv^2,$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{(m/2) + (I/2r^2) + (M/3)}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}\ .$$

With $M = 4.5$ kg, $m = 0.60$ kg, $r = 5.0$ cm, $h = 0.82$ m, and $I = 3.0\times10^{-3}$ kg m^2, we have $v = 1.4$ m/s.

78.• We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2 and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three eqs..

$$T_1 - m_1g = m_1a_1,\ m_2g - T_2 = m_2a_2,\ T_2R - T_1R = I\alpha.$$

(**a**) The rotational inertia of the disk is $I = (½)MR^2$ (Table 10-2(c)), so we divide the third eq. (above) by R, add them all, and use the earlier equality among accelerations — to obtain:

$$m_2g - m_1g = [m_1+m_2+(½)M]a,$$

which yields $a = 4g/25 = 1.57$ m/s^2. (**b**) Plugging back in to the first eq., we find T_1 = 29m_1g/25 = 4.55 N, where it is important in this step to have the mass in SI units: m_1 = 0.40 kg. (**c**) Similarly, with m_2 = 0.60 kg, we find T_2 = 5m_2g/6 = 4.94 N.

84.• We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by (½)L, where L is the length of any one of the rods. The gravitational potential energy decreases by (½)MgL, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written (½)$I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus,

$$0 = -\frac{1}{2}MgL + \frac{1}{2}I\omega^2 \Rightarrow \quad \omega = \sqrt{\frac{MgL}{I}}\,.$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is $I = (ML^2/3) + (ML^2/9) = 4ML^2/9$.
Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} = \sqrt{\frac{9(9.800\ \mathrm{m/s^2})}{4(0.600\ \mathrm{m})}}$$
$$= 6.06\ \mathrm{rad/s}.$$

89.• The center of mass is initially at height h = (½)Lsin40° when the system is released (where L = 2.0 m). The corresponding potential energy Mgh (where M = 1.5 kg) becomes rotational kinetic energy (½)$I\omega^2$ as it passes the horizontal position (where I is the rotational inertia about the pin). Using Table 10-2 (e) and the parallel axis theorem, we find

$$I = \tfrac{1}{2}ML^2 + M(\tfrac{1}{2}L)2 = \tfrac{1}{3}ML^2.$$

Therefore, $Mg\tfrac{1}{2}L\sin 40° = \tfrac{1}{2}(\tfrac{1}{3}ML^2)\omega^2$

$$\Rightarrow \omega = \sqrt{3g\sin 40°/L} = 3.1\ \text{(rad/s)}.$$

125.• The mass of the Earth is M = 5.98×10^{24} kg and the radius is R = 6.37×10^6 m. (**a**) Assuming the Earth to be a sphere of uniform density, its moment of inertia is

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(5.98\times10^{24})(6.37\times10^6)$$
$$= 9.71\times10^{37}\ \mathrm{kg\cdot m^2}.$$

(**b**) The angular speed of the Earth is $\omega = 2\pi/T$ = 2π/86400s = 7.27×10^{-5}rad/s. Thus, its rotational kinetic energy is

$$K_{\rm rot} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(9.71\times10^{37})(7.27\times10^{-5})^2$$
$$= 2.57\times10^{29}\ \text{(J)}.$$

(**c**) The amount of time the rotational energy could be supplied to at a rate of P = 1.0 kW = 1.0×10^3 J/s to a population of approximately N = 5.0×10^9 people is

$$\Delta t = \frac{K_{rot}}{NP} = \frac{2.57\times10^{29}\ \mathrm{J}}{(5.0\times10^9)(1.0\times10^3\ \mathrm{J/s})}$$
$$= 5.14\times10^{16}\ \mathrm{s} \sim 1.6\times10^9\ \mathrm{yr}.$$

83.• With rightward positive for the block and clock- wise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P-T = ma$, where m = 2.0 kg. Applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$, where I = 0.050 kg·m^2. Noting that $R\alpha = a_t = -a$, we multiply this eq. by R and obtain

$$-TR^2 = -Ia \quad \Rightarrow \quad T = aI/R^2.$$

Adding this to the above eq. (for the block) leads to $P = (m+I/R^2)a$. Thus, a = 0.92 m/s^2 and therefore α = –4.6 rad/s^2 (or $|\alpha|$ = 4.6 rad/s^2), where the negative sign in α should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

重點整理－第 10 章 轉動

什麼原因造成雲霄飛車頭痛症？

角位置 爲了描述剛體繞**定軸**(稱爲**轉軸**)轉動，可假定一固定於剛體內參考線，其垂直轉軸並與剛體一起轉動。量測此線相對於固定方向的角位置θ，當θ以弧度計量時，

$$\theta = s/r \quad \text{(弧度計量)}, \qquad (10\text{-}1)$$

式中 s 爲一半徑 r 及角度θ的圓弧長。弧度計量使轉數和度數之關係爲

$$1\ \text{rev} = 360° = 2\pi\ \text{rad}. \qquad (10\text{-}2)$$

角位移 物體繞轉軸轉動，角位置由θ_1改變至θ_2時，歷經一段角位移

$$\Delta\theta = \theta_2 - \theta_1, \qquad (10\text{-}4)$$

式中對逆時鐘轉$\Delta\theta$爲正，而對順時鐘轉爲負。

角速度與角速率 若物體於時距Δt 內轉動角位移$\Delta\theta$，其平均角速度ω_{av}爲

$$\omega_{av} = \Delta\theta/\Delta t\,. \qquad (10\text{-}5)$$

物體(瞬時)角速度ω爲

$$\omega = d\theta/dt\,. \qquad (10\text{-}6)$$

ω_{av}和ω兩者皆爲向量，其方向由圖 10-6 的**右手定則**給定。對逆時鐘轉動其爲正而順時鐘轉動爲負。物體的角速度大小則爲角速率。

角加速度 若物體的角速度於時距$\Delta t = t_2 - t_1$內從ω_1變化至ω_2，物體的**平均角加速度**α_{av}爲

$$\alpha_{av} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}. \qquad (10\text{-}7)$$

物體(瞬時)**角加速度**α爲

$$\alpha = d\omega/dt\,, \qquad (10\text{-}8)$$

α_{av} 和α兩者皆爲都是向量。

等角加速度的運動學方程式 等角加速度(α = 常數)爲轉動運動中重要的特例，適當的運動學方程式於表 10-1 給定，其爲 (10-12/13/16)

$$\omega-\omega_0=\alpha t,\ \theta-\theta_0=\omega_0 t+\tfrac{1}{2}\alpha t^2=\omega t-\tfrac{1}{2}\alpha t^2,$$

$$\omega^2-\omega_0^2=2\alpha(\theta-\theta_0),\ \theta-\theta_0=\tfrac{1}{2}(\omega+\omega_0)t, \quad (10\text{-}14/15)$$

線變數與角變數之關係 剛性轉體內至轉軸垂直距離 r 的點，於半徑爲 r 之圓周上運動。若物體轉動θ角度，該點沿著圓弧運動的距離 s 爲

$$s=\theta\, r \quad (\text{弧度計量}), \quad (10\text{-}17)$$

式中θ以徑度爲單位。此點的線速度 $\vec{v}$ 與圓正切，其線速率爲

$$v=\omega r \quad (\text{弧度計量}), \quad (10\text{-}18)$$

式中ω爲物體的角速度(單位爲每秒之弧度 rad/s)。該點的線加速度 $\vec{a}$ 有切向與徑向分量，切向分量爲

$$a_t=\alpha r \quad (\text{弧度計量}), \quad (10\text{-}22)$$

式中α爲物體的角加速度大小(單位爲每秒平方之弧度 rad/s^2)，而徑向分量爲

$$a_r=v^2/r=\omega^2 r \quad (\text{弧度計量}). \quad (10\text{-}23)$$

若該點作等速率圓周運動，對於點及物體其運動周期 T 爲

$$T=2\pi r/v=2\pi/\omega \quad (\text{弧度計量}). \quad (10\text{-}19,20)$$

轉動動能與轉動慣量 繞定軸轉動的剛體之動能爲

$$K_{\text{rot}}=\tfrac{1}{2}I\omega^2 \quad (\text{弧度計量}), \quad (10\text{-}34)$$

其中 I 爲物體的轉動慣量，對於不連續質點組成的系統其定義爲

$$I=\sum m_i r_i^2, \quad (10\text{-}33)$$

而對於連續質量分佈的物體則其定義爲

$$I=\int r^2\,dm, \quad (10\text{-}35)$$

其中 r 和 r_i 在這些式中代表物體中各質量元至轉軸的垂直距離。

平行軸定理 物體繞著任意軸轉動之轉動慣量 I 與其繞著通過質心的平行軸轉動之轉動慣量 I_{cm} 之關係爲**平行軸定理**：

$$I=I_{\text{cm}}+Mh^2 \quad (10\text{-}36)$$

式中 h 爲兩軸的垂直距離。

力矩 力矩爲由力 $\vec{F}$ 造成物體繞著轉軸旋轉或扭曲之作用。若力 $\vec{F}$ 施於相對於軸的位置向量 $\vec{r}$ 的點，則力矩大小爲

$$\tau=rF_t=r_\perp F=rF\sin\phi, \quad (10\text{-}40,41,39)$$

式中 F_t 爲 $\vec{F}$ 垂直 $\vec{r}$ 的分量，而 ϕ 爲 $\vec{r}$ 與 $\vec{F}$ 的夾角；$r_\perp$爲轉軸與通過 $\vec{F}$ 向量的延伸線之垂直距離，此線稱爲 $\vec{F}$ 的**作用線**，而 $r_\perp$稱爲 $\vec{F}$ 的**力臂**；同理，r 爲 F_t的力臂。力矩的單位(SI 制)爲 N·m。若力矩傾向於逆時鐘方向轉動靜止的物體爲正，而順時鐘方向者爲負。

角式之牛頓第二定律 牛頓第二定律的轉動類比爲

$$\tau_{net}=I\alpha, \quad (10\text{-}45)$$

其中τ_{net}爲作用於質點或剛體上的淨力矩，I 爲質點或物體對轉軸的轉動慣量，而α爲對該轉軸產生的角加速度。

功與轉動動能 用以計算轉動運動中的功與功率的方程式對應於用於平移運動的方程式，其爲

$$W=\int_{\theta_i}^{\theta_f}\tau\,d\theta, \quad (10\text{-}53)$$

及

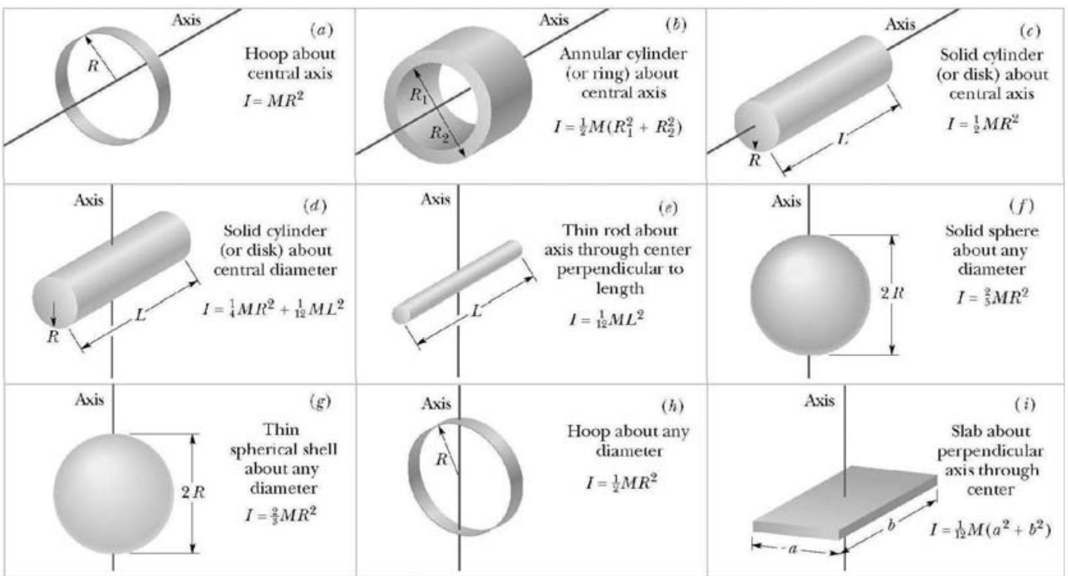
$$P=dW/dt=\tau\omega. \quad (10\text{-}55)$$

當 τ 爲定值時，10-53 式簡化成

$$W=\tau(\theta_f-\theta_i). \quad (10\text{-}55)$$

用於轉動體之功–動能定理形式爲

$$\Delta K=K_f-K_i=\tfrac{1}{2}I\omega_f^2-\tfrac{1}{2}I\omega_i^2=W. \quad (10\text{-}52)$$



•備忘錄•

rigid body 剛體;translation 平移;rotation 轉動;fixed axis 定軸; rotation axis 轉軸; angular position/displacement/velocity/acceleration 角位置/位移/速度/加速度; radian, 弧度/徑;variable 變數; parallel-axis theorem 平行軸定理; rotational inertia 轉動慣量;torque 力矩;line of action 作用線; moment arm 力臂; cylinder 圓柱;hoop 環/戒指; sphere 球; beverage 飲料;vertigo 眩暈;skull 頭蓋骨;vein 血管; figure skater 花式溜冰; roller coaster 雲霄飛車; judo 柔道; hip throw 腰摔??;

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Ex.5-1: Prob.10-31.

Prob. *11-1, 5, 8, 15, 21, 24, 31, 37, 42, 51, 60, 64, 67, 72, 77, 81, 85 (tentatively)*

124.• (**a**) The angular speed ω associated with Earth's spin is $\omega = 2\pi/T$, where T = 86400 s (one day). Thus $\omega = 2\pi/86400$ s $= 7.3\times10^{-5}$ rad/s, and the angular acceleration α required to accelerate the Earth from rest to ω in one day is $\alpha = \omega/T$. The torque needed is then $\tau = I\alpha = I\omega/T = (9.7\times10^{37})(7.3\times10^{-5})/86400 = 8.2\times10^{28}$ (N·m), where we used $I = (2/5)MR^2$ for Earth's rotational inertia. (**b**) Using the values from part (a), the kinetic energy of the Earth associated with its rotation about its own axis is $K = (½)I\omega^2 = 2.57\times10^{29}$ J. This is how much energy would need to be supplied to bring it (starting from rest) to the current angular speed. (**c**) The associated power is

$$P = K/T = 2.57\times10^{29}\text{ J}/ 86400\text{ s} = 2.0\times10^{24}\text{ W}.$$