Chapter **9** *Center of Mass and Linear Momentum*

01. Our notation is as follows: $x_1 = 0$ and $y_1 = 0$ are the coordinates of the $m_1 = 3.0$ kg particle; $x_2 = 2.0$ m and $y_2 = 1.0$ m are the coordinates of the $m_2 = 4.0$ kg particle; and, $x_3 = 1.0$ m and $y_3 = 2.0$ m are the coordinates of the $m_3 = 8.0$ kg particle. (**a**) With $M = m_1+m_2+m_3 = 15.0$ kg, the x coordinate of the center of mass is $x_{cm} = (m_1x_1+m_2x_2+m_3x_3)/M$ = [0+(4.0 kg)(2.0 m)+(8.0 kg)(1.0 m)]/15.0 kg = 1.1 m. (**b**) The y coordinate of the center of mass is $y_{cm} = (m_1y_1+m_2y_2+m_3y_3)/M$ = [0+(4.0 kg)(1.0 m)+ (8.0 kg)(2.0 m)]/15.0 kg = 1.3 m. (**c**) As the mass of m_3, the topmost particle, is increased, the center of mass shifts toward that particle. As we approach the limit where m_3 is infinitely more massive than the others, the center of mass becomes infinitesimally close to the position of m_3.

03. We will refer to the arrangement as a "table." We locate the coordinate origin at the left end of the tabletop (as shown in Fig. 9-37). With $+x$ rightward and $+y$ upward, then the center of mass of the right leg is at $(x, y) = (+L, -L/2)$, the center of mass of the left leg is at $(x, y) = (0, -L/2)$, and the center of mass of the tabletop is at $(x, y) = (L/2, 0)$. (**a**) With $M_t = M+M+3M = 5M$, the x coordinate of the (whole table) center of mass is $x_{cm} = [ML + M(0) + 3M(L/2)]/M_t = 0.5L$. With $L = 22$ cm, we have $x_{cm} = 11$ cm. (**b**) The y coordinate of the (whole table) center of mass is $y_{cm} = [M(-L/2) + M(-L/2) + 3M(0)]/M_t = -L/5$, or $y_{cm} = -4.4$ cm. From the coordinates, we see that the whole table center of mass is a small distance 4.4 cm directly below the middle of the tabletop.

05. (**a**) By symmetry the center of mass is located on the axis of symmetry of the molecule – the y axis. Therefore $x_{cm} = 0$. (**b**) To find y_{cm}, we note that $3m_H y_{cm} = m_N(y_N - y_{cm})$, where y_N is the distance from the nitrogen atom to the plane containing the three hydrogen atoms:

$$y_N = \sqrt{(10.14\times10^{-11})^2 - (9.4\times10^{-11})^2} = 3.803\times10^{-11}\ (\text{m}).$$

Thus,
$$y_{cm} = \frac{m_N y_N}{m_N + 3m_H} = \frac{(14.0067)(3.803\times10^{-11})}{(14.0067)+3(1.00797)} = 3.13\times10^{-11}\ (\text{m}).$$

where Appendix F has been used to find the masses.

15. We need to find the coordinates of the point where the shell explodes and the velocity of the fragment that does not fall straight down. The coordinate origin is at the firing point, the $+x$ axis is rightward, and the $+y$ direction is upward. The y component of the velocity is given by $v = v_{0y} - gt$ and this is zero at time $t = v_{0y}/g = (v_0/g)\sin\theta_0$, where v_0 is the initial speed and θ_0 is the firing angle. The coordinates of the highest point on the trajectory are

$$x = v_{ox}t = v_o t\cos\theta = \frac{v_0^2}{2g}\sin 2\theta_0 = \frac{20^2}{2(9.8)}\sin 120° = 17.7\ (\text{m}).$$

and
$$y = v_{0y}t - \tfrac{1}{2}gt^2 = \frac{v_0^2}{2g}\sin^2\theta_0 = \frac{20^2}{2(9.8)}\sin^2 60° = 15.3\ (\text{m}).$$

Since no horizontal forces act, the horizontal component of the momentum is conserved. Since one fragment has a velocity of zero after the explosion, the momentum of the other equals the momentum of the shell before the explosion. At the highest point the velocity of the shell is $v_0\cos\theta_0$, in the positive x direction. Let M be the mass of the shell and let V_0 be the velocity of the fragment. Then $Mv_0\cos\theta_0 = MV_0/2$, since the mass of the fragment is $M/2$. This means

$$V_0 = 2\,v_0\cos\theta_0 = 2\,(20\text{ m/s})\cos 60° = 20\text{ m/s}.$$

This information is used in the form of initial conditions for a projectile motion problem to determine where the fragment lands. Resetting our clock, we now analyze a projectile launched horizontally at time $t = 0$ with a speed of 20 m/s from a location having coordinates $x_0 = 17.7$ m, $y_0 = 15.3$ m. Its y coordinate is given by $y = y_0 - (½)gt^2$ and when it lands this is zero. The time of landing is $t = (2y_0/g)^{1/2}$ and the x coordinate of the landing point is

$$x = x_o + V_o t = x_o + V_o\sqrt{2y_0/g} = 17.7 + (20)\sqrt{2(15.3)/9.8} = 53\ (\text{m}).$$

17. There is no net horizontal force on the dog-boat system, so their center of mass does not move. Therefore by Eq. 9-16, $M\Delta x_{cm} = 0 = m_b\Delta x_b + m_d\Delta x_d$, which implies $|\Delta x_b| = (m_d/m_b)|\Delta x_d|$. Now we express the geometrical condition that *relative to the boat* the dog has moved a distance $d = 2.4$ m: $|\Delta x_b| + |\Delta x_d| = d$, which accounts for the fact that the dog moves one way and the boat moves the other. We substitute for $|\Delta x_b|$ from above:

$$\frac{m_d}{m_b}|\Delta x_d| + |\Delta x_d| = d,$$

which leads to $|\Delta x_d| = d/(1+m_d/m_b) = 2.4/(1+4.5/18) = 1.92$ (m). The dog is therefore 1.9 m closer to the shore than initially (where it was $D = 6.1$ m from it). Thus, it is now $D - |\Delta x_d| = 4.2$ m from the shore.

20. (**a**) Since the force of impact on the ball is in the y direction, p_x is conserved:

$$p_{x\,i} = mv_i\sin\theta_i = p_{x\,f} = mv_f\sin\theta_2\,.$$

With $\theta_i = 30.0°$ and $v_i = v_f$, we find $\theta_2 = 30.0°$. (**b**) The momentum change is

$$\Delta\vec{p} = mv_f\cos\theta_2(-\hat{\text{j}}) - mv_i\cos\theta_1(+\hat{\text{j}})$$

$$= -2\,(0.165\text{ kg})(2.00\text{ m/s})\cos 30°(\hat{\text{j}})$$
$$= (-0.572\text{ kg}\cdot\text{m/s})(\hat{\text{j}}).$$

26. We choose $+y$ upward, which means $v_i = -25$ m/s and $v_f = +10$ m/s. During the collision, we make the reasonable approximation that the net force on the ball is equal to F_{av} – the average force exerted by the floor up on the ball. (**a**) Using the impulse momentum theorem (Eq. 9-31) we find

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = (1.2)(10)(\hat{\text{j}}) - (1.2)(-25)(\hat{\text{j}})$$
$$= 42\ (\text{kg·m/s})\ \hat{\text{j}}.$$

(**b**) From Eq. 9-35, we obtain

$$\vec{F}_{av} = \vec{J}/\Delta t = 42\,\hat{\text{j}}/0.020 = 2.1\times10^3(\text{N})\,\hat{\text{j}}.$$

29. We use coordinates with $+x$ rightward and $+y$ upward, with the usual conventions for measuring the angles (so that the initial angle becomes 180°+ 35° = 215°). Using SI units and magnitude-angle notation (efficient to work with when using a vector-capable calculator), the change in momentum is

$$\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (3.00\text{ kg·m/s}\angle 90°)$$
$$-\ (3.60\text{ kg·m/s}\angle 215°) = 5.86\text{ kg·m/s}\angle 59.8°.$$

(**a**) The magnitude of the impulse is $J = \Delta p = 5.86$ kg·m/s. (**b**) The direction of $\boldsymbol{J}$ is 59.8° measured counterclockwise from the $+x$ axis. (**c**) Eq. 9-35 leads to

$$J = F_{av}\,\Delta t = 5.86 \Rightarrow F_{av} = 5.86/(2.00\times10^{-3})$$
$$\approx 2.93\times10^3\ (\text{N}).$$

We note that this force is very much larger than the weight of the ball, which justifies our (implicit) assumption that gravity played no significant role in the collision. (**d**) The direction of $\boldsymbol{F}_{av}$ is the same as $\boldsymbol{J}$, 59.8° measured counterclockwise from the $+x$ axis.

33. From Fig. 9-55, $+y$ corresponds to the direction of the rebound (directly away from the wall) and $+x$ towards the right. Using unit-vector notation, the ball's initial and final velocities are

$$\vec{v}_i = v\cos\theta\,\hat{\text{i}} - v\sin\theta\,\hat{\text{j}} = 5.2\,\hat{\text{i}} - 3.0\,\hat{\text{j}}$$

and $$\vec{v}_f = v\cos\theta\,\hat{\text{i}} + v\sin\theta\,\hat{\text{j}} = 5.2\,\hat{\text{i}} + 3.0\,\hat{\text{j}},$$

respectively (with SI units understood). (**a**) With $m = 0.30$ kg, the impulse-momentum theorem (Eq. 9-31) yields

$$\vec{J} = m\vec{v}_f - m\vec{v}_i = 2\,(0.30\text{ kg})(3.0\text{ m/s})\,(\hat{\text{j}}) = 1.8\text{ N.s}\,\hat{\text{j}}.$$

(**b**) Using Eq. 9-35, the force on the ball by the wall is $\boldsymbol{J}/\Delta t = (1.8/0.010)\mathbf{j} = (180\text{ N})\mathbf{j}$. By Newton's third law, the force on the wall by the ball is $(-180\text{ N})\mathbf{j}$ (that is, its magnitude is 180 N and its direction is directly into the wall, or "down" in the view provided by Fig. 9-55).

44. This problem involves both mechanical energy conservation $U_i = U_{sp} = K_1 + K_2$, where $U_i = 60$ J, and momentum conservation $0 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$, where $m_2 = 2m_1$. From the second eq., we find $|\boldsymbol{v}_1| = 2|\boldsymbol{v}_2|$, which in turn implies (since $v_1 = |\boldsymbol{v}_1|$ and likewise for $\boldsymbol{v}_2$) <*cf. Prob.77*>

$$K_1 = \tfrac{1}{2}m_1v_1^2 = (½)(½m_2)(2v_2)^2 = 2K_2.$$

(**a**) We substitute $K_1 = 2K_2$ into the energy conservation relation and find

$$U_i = 2K_2 + K_2 \quad\Rightarrow\quad K_2 = U_i/3 = 20\text{ J}.$$

(**b**) And we obtain $K_1 = 2K_2 = 2(20) = 40$ (J).

46. We refer to the discussion in the textbook (see S. P. 9-8, which uses the same notation that we use here) for many of the important details in the reasoning. Here we only present the primary computational step (using SI units):

$$v = \frac{m+M}{m}\sqrt{2gh} = \frac{0.010+2.0}{0.010}\sqrt{2\,(9.80)(0.12)}$$
$$= 3.1\times10^2\ (\text{m/s}).$$

50. We think of this as having two parts: the first is the collision itself – where the bullet passes through the block so quickly that the block has not had time to move through any distance yet – and then the subsequent "leap" of the block into the air (up to height h measured from its initial position). The first part involves momentum conservation (with $+y$ upward):

$$(0.01\text{ kg})(1000\text{ m/s}) = (5.0\text{ kg})\ \underline{v}$$
$$+\ (0.01\text{ kg})(400\text{ m/s}),$$

which yields $\underline{v} = 1.2$ m/s. The second part involves either the free-fall eqs. from Ch. 2 (since we are ignoring air friction) or simple energy conservation from Ch.8. Choosing the latter approach, we have $(½)(5.0\text{ kg})(1.2\text{ m/s})^2 = (5.0\text{ kg})\,(9.80\text{ m/s}^2)h$, which gives the result $h = 0.073$ m.

53. As hinted in the *Pb.* statement, the velocity v of the system as a whole – when the spring reaches the maximum compression x_m – satisfies

$$m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)\,v.$$

The change in kinetic energy of the system is therefore

$$\Delta K = \tfrac{1}{2}(m_1+m_2)v^2 - \tfrac{1}{2}m_1v_{1i}^2 - \tfrac{1}{2}m_2v_{2i}^2$$
$$= \frac{(m_1v_{1i} + m_2v_{2i})^2}{2(m_1+m_2)} - \frac{1}{2}m_1v_{1i}^2 - \frac{1}{2}m_2v_{2i}^2,$$

which yields $K = -35$ J. (Although it is not necessary to do so, still it is worth noting that algebraic manipulation of the above expression leads to $\Delta K = (½)(m_1m_2)v_{rel}^2/(m_1+m_2)$, where $v_{rel} = v_1 - v_2$). Conservation of energy then requires

$$\frac{1}{2}kx_m^2 = -\Delta K \Rightarrow x_m = \sqrt{\frac{-2\Delta K}{k}} = \sqrt{\frac{-2(-35)}{1120}}$$
$$= 0.25\ (\text{m}).$$

56. (**a**) Let m_A be the mass of the block on the left, v_{Ai} be its initial velocity, and v_{Af} be its final velocity. Let m_B be the mass of the block on the right, v_{Bi} be its initial velocity, and v_{Bf} be its final velocity. The momentum of the two-block system is conserved, so

$$m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf}$$

and $$v_{Af} = \frac{m_A v_{Ai} + m_B v_{Bi} - m_B v_{Bf}}{m_A}$$

$$= \frac{(1.6)(5.5) + (2.4)(2.5) - (2.4)(4.9)}{1.6} = 1.9 \text{ (m/s).}$$

(**b**) The block continues going to the right after the collision. (**c**) To see if the collision is elastic, we compare the total kinetic energy before the collision with the total kinetic energy after the collision. The total kinetic energy before is

$$K_i = \tfrac{1}{2} m_A v_{Ai}{}^2 + \tfrac{1}{2} m_B v_{Bi}{}^2 = \tfrac{1}{2}(1.6)(5.5)^2 + \tfrac{1}{2}(2.4)(2.5)^2 = 31.7 \text{ (J).}$$

The total kinetic energy after is

$$K_f = \tfrac{1}{2} m_A v_{Af}{}^2 + \tfrac{1}{2} m_B v_{Bf}{}^2 = \tfrac{1}{2}(1.6)(1.9)^2 + \tfrac{1}{2}(2.4)(4.9)^2 = 31.7 \text{ (J).}$$

Since $K_i = K_f$ the collision is found to be *elastic*.

60. First, we find the speed v of the ball of mass m_1 right before the collision (just as it reaches its lowest point of swing). Mechanical energy conservation (with h = 0.700 m) leads to

$$m_1 gh = \tfrac{1}{2} m_1 v^2 \quad \Rightarrow \quad v = \sqrt{2gh} = 3.7 \text{ m/s.}$$

(**a**) We now treat the elastic collision (with SI units) using Eq. 9-67:

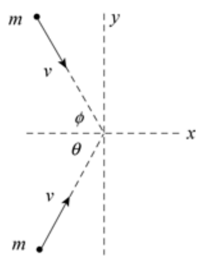
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v = \frac{0.5 - 2.5}{0.5 + 2.5}(3.7) = -2.47 \text{ (m/s),}$$

which means the final speed of the ball is 2.47 m/s . (**b**) Finally, we use Eq. 9-68 to find the final speed of the block:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v = \frac{2(0.5)}{0.5 + 2.5}(3.7) = 1.23 \text{ (m/s).}$$

69. Suppose the objects enter the collision along lines that make the angles $\theta > 0$ and $\phi > 0$ with the x axis, as shown in the diagram that follows. Both have the same mass m and the same initial speed v. We suppose that after the collision the combined object moves in the positive x direction with speed V. Since the y component of the total momentum of the two-object system is conserved,

$$mv \sin\theta - mv \sin\phi = 0.$$



This means $\theta = \phi$. Since the x component is conserved, $2mv \cos\theta = 2mV$. We now use $V = v/2$ to find that $\cos\theta = 1/2$. This means $\theta = 60°$. The angle between the initial velocities is 120°.

71. (**a**) The thrust of the rocket is given by $T = R\, v_{\text{rel}}$ where R is the rate of fuel consumption and v_{rel} is the speed of the exhaust gas relative to the rocket. For this problem R = 480 kg/s and $v_{\text{rel}} = 3.27\times10^3$ m/s, so T = (480 kg/s)(3.27×10^3 m/s) = 1.57×10^6 N. (**b**) The mass of fuel ejected is given by $M_{\text{fuel}} = R\Delta t$ where Δt is the time interval of the burn. Thus, M_{fuel} = (480 kg/s)(250 s) = 1.20×10^5 kg. The mass of the rocket after the burn is $M_f = M_i - M_{\text{fuel}}$ = (2.55×10^5 kg) − (1.20×10^5 kg) = 1.35×10^5 kg. (**c**) Since the initial speed is zero, the final speed is given by

$$v_f = v_{\text{rel}} \ln\frac{M_i}{M_f} = (3.27\times10^3)\ln\frac{2.55\times10^5}{1.35\times10^5} = 2.08\times10^3 \text{ (m/s).}$$

109.• (**a**) Let v be the final velocity of the ball-gun system. Since the total momentum of the system is conserved $mv_i = (m+M)\, v$. Therefore,

$$v = mv_i/(m+M) = (60 \text{ g})(22 \text{ m/s})/(60 \text{ g}+240 \text{ g}) = 4.4 \text{ m/s.}$$

(**b**) The initial kinetic energy is $K_i = (½)mv_i^2$ and the final kinetic energy is $K_f = (½)(m+M)v^2 = (½)m^2v_i^2/(m+M)$. The problem indicates $\Delta E_{th} = 0$, so the difference $K_i - K_f$ must equal the energy U_s stored in the spring:

$$U_s = \frac{1}{2}mv_i^2 - \frac{1}{2}\frac{(mv_i)^2}{m+M} = \frac{1}{2}mv_i^2\left(1 - \frac{m}{m+m}\right) = \frac{1}{2}mv_i^2\,\frac{M}{m+m}.$$

Consequently, the fraction of the initial kinetic energy that becomes stored in the spring is

$$\frac{U_s}{K_i} = \frac{M}{m+M} = \frac{240}{60+240} = 0.80 .$$

131.• The mass of each ball is m, and the initial speed of one of the balls is v_{1i} = 2.2 m/s. We apply the conservation of linear momentum to the x and y axes, respectively.

$$m\, v_{1i} = mv_{1f}\cos\theta_1 + mv_{2f}\cos\theta_2$$

and $$0 = mv_{1f}\sin\theta_1 - mv_{2f}\sin\theta_2 .$$

The mass m cancels out of these eqs., and we are left with two unknowns and two eqs., which is sufficient to solve. (**a**) The y-momentum eq. can be rewritten as, using $\theta_2 = 60°$ and v_{2f} = 1.1 m/s,

$$mv_{1f}\sin\theta_1 = (1.1 \text{ m/s}) \sin 60° = 0.95 \text{ m/s,}$$

and the x-momentum eq. yields

$$mv_{1f}\cos\theta_1 = (2.2 \text{ m/s}) - (1.1 \text{ m/s})\cos 60° = 1.65 \text{ m/s.}$$

Dividing these two eqs., we find $\tan\theta_1$ = 0.576 which yields $\theta_1 = 30°$. We plug the value into either eq. and find v_{1f} = 1.9 m/s. (**b**) From the above, we have $\theta_1 = 30°$. (**c**) One can check to see if this an elastic collision by computing

$$\frac{2K_i}{m} = v_{1i}^2 \quad \text{and} \quad \frac{2K_f}{m} = v_{1i}^2 + v_{2f}^2$$

and seeing if they are equal (they are), but one must be careful not to use rounded-off values. Thus, it is useful to note that the answer in part (a) can be expressed "exactly" as $v_{1f} = v_{1i}(3)^{1/2}/2$ (and of course

$v_{2f} = v_{1i}/2$ "exactly" — which makes it clear that these two kinetic energy expressions are indeed equal).

133.• (**a**) We locate the coordinate origin at the center of Earth. Then the distance r_{cm} of the center of mass of the Earth-Moon system is given by $r_{cm} = m_M r_M/(m_M+m_E)$, where m_M is the mass of the Moon, m_E is the mass of Earth, and r_M is their separation. These values are given in Appendix C. The numerical result is

$$r_{cm} = \frac{7.36\times10^{22}}{7.36\times10^{22}+5.98\times10^{24}}(3.82\times10^{8})$$
$$= 4.64\times10^{6} = 4.6\times10^{3}\ (\text{km}).$$

(**b**) The radius of Earth is $R_E = 6.37\times10^6$ m, so $r_{cm}/R_E = 0.73 = 73\%$.

135.• (**a**) The thrust is $R\,v_{rel}$ where $v_{rel} = 1200$ m/s. For this to equal the weight Mg where $M = 6100$ kg, we must have $R = (6100)(9.8)/1200 \approx 50$ kg/s. (**b**) Using Eq. 9-42 with the additional effect due to gravity, we have $R\,v_{rel} - Mg = Ma$, so that requiring $a = 21$ m/s^2 leads to $R = (6100)(9.8+21)/1200 = 1.6\times10^2$ (kg/s).

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

center of mass 質心; (conservation of) linear momentum 線動量(守恆);system of particles 質點系統; closed system 封閉的系統;inelastic/elastic collision 非彈性/彈性碰撞;impulse 衝量;ballistic pendulum 衝擊擺; ballet 芭蕾舞;grand jeté 大踼腿;glancing 掠入的;give 彈性,伸展性; racetrack 賽車道;rocket 火箭; thrust 推力;solid body 實心體; variable-mass 變質量

重點整理—第 9 章 質心與線動量

假如這事實是真的，那是如何成真？

質心 n個質點的系統之質心定義爲一點，其座標爲下式給定

$$x_{cm} = M^{-1}\sum_{i=1}^{n} m_i x_i\ ,\ y_{cm} = M^{-1}\sum_{i=1}^{n} m_i y_i\ ,$$
$$z_{cm} = M^{-1}\sum_{i=1}^{n} m_i z_i\ , \qquad (9\text{-}5)$$

或 $$\vec{r} = M^{-1}\sum_{i=1}^{n} m\vec{r}_i\ , \qquad (9\text{-}8)$$

式中M爲系統總質量。

質點的系統之牛頓第二定律 任意質點系統的質心運動受牛頓第二定律支配，其爲

$$\vec{F}_{net} = M\,\vec{a}_{cm}\ , \qquad (9\text{-}14)$$

$\vec{F}_{net}$ 爲所有作用於系統外力之淨力，M 爲系統總質量，而 $\vec{a}_{cm}$ 爲系統質心之加速度。

線動量與牛頓第二定律 對於單一質點，線動量 $\vec{p}$ 定義如下 $$\vec{p} = m\vec{v}\ . \qquad (9\text{-}22)$$

利用線動量可將牛頓第二定律寫爲

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}\ . \qquad (9\text{-}23)$$

對於質點系統，這些關係式變爲

$$\vec{P} = M\,\vec{v}_{cm}\ \text{及}\ \vec{F}_{net} = \frac{d\vec{P}}{dt}\ . \qquad (9\text{-}25,9\text{-}27)$$

碰撞與衝量 運用動量型式之牛頓第二定律於涉及碰撞之似質點物體導得**衝量－線動量定理**：

$$\vec{p}_f - \vec{p}_i = \Delta\vec{p} = \vec{J}\ , \qquad (9\text{-}31,\ 9\text{-}32)$$

式中 $\vec{p}_f - \vec{p}_i = \Delta\vec{p}$ 爲物體之線動量改變量，$\vec{J}$ 爲其他物體施於此物體之力 $\vec{F}(t)$ 所造成的衝量

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)dt\ . \qquad (9\text{-}30)$$

假如於碰撞過程，作用力 $\vec{F}(t)$ 的平均大小爲 F_{av}，而Δt 爲碰撞持續時間，那麼對於一維運動

$$J = F_{av}\Delta t. \qquad (9\text{-}35)$$

當一穩定的物體流(各物體皆具質量 m 和速率 v)與固定的物體碰撞，對此固定體之平均作用力爲

$$F_{av} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t}m\Delta v\ , \qquad (9\text{-}37)$$

式中 $n/\Delta t$ 爲物體與固定體撞擊的時變率，Δv 爲各撞擊物體的速度改變量，其平均作用力可寫成

$$F_{av} = -\frac{\Delta m}{\Delta t}\Delta v\ , \qquad (9\text{-}40)$$

其中$\Delta m/\Delta t$ 爲撞擊固定體的質量時變率，於 9-37 及 9-40 式中，若物體撞擊後即停止，則物體速度變化量$\Delta v = -v$；若其以不變的速率直接往後反彈，則$\Delta v = -2v$。

線動量守恆 假如系統爲孤立的，其所受外力總和爲零，則系統的線動量 $\boldsymbol{P}$ 維持不變

$$\vec{P} = \text{常數}\ (\text{封閉且孤立系統}). \qquad (9\text{-}42)$$

上式亦可寫成

$$\vec{P}_i = \vec{P}_f\ , \qquad (9\text{-}43)$$

式中下標 i 及 f 提及某一起始時間及稍後時間之動量值，(9-42)及(9-43)式爲**線動量守恆定律**之等同敘述。

一維非彈性碰撞 於兩物體一維非彈性碰撞中，這二體系統的動能不守恆。若系統爲封閉且孤立的，則系統線動量必守恆，寫成向量型式爲

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}\ , \qquad (9\text{-}50)$$

式中 i、f 提及恰碰撞前和恰碰撞後的的線動量值。假如物體運動係沿著某單一軸，碰撞爲一維的，即可利用速度沿該軸分量將 9-50 式寫成

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}. \qquad (9\text{-}51)$$

若物體碰撞後黏合一起，這種碰撞爲**完全非彈性碰撞**，兩物體就擁有相同的末速度 v (因其黏在一起)。

質心運動 封閉且孤立的兩碰撞物體系統之質心不受碰撞影響，特別地質心速度亦不受碰撞改變。

一維彈性碰撞 彈性碰撞爲一特殊碰撞型態，其中碰撞體系統之動能守恆。若系統爲封閉且孤立的，其線動量亦守恆。對於一維碰撞，其中物體 2 爲定靶而物體 1 爲入射的投射體，由動能與線動量守恆得到碰撞後的速度表示式

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}, \qquad (9\text{-}67)$$

及

$$v_{2f} = \frac{2m_i}{m_1 + m_2} v_{1i}. \qquad (9\text{-}68)$$

然而對於物體 2 爲動靶的碰撞，則

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \qquad (9\text{-}75)$$

及

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{1i}, \qquad (9\text{-}76)$$

- 質心：均質細棍/長方形板–幾何中心；均質三角形–三中線交點；均質實心(中空)圓錐–對稱軸上離錐頂 3/4 (2/3)高度處。***Q.***兩等長臂之 V 字形的質心？
- 火箭介紹請參考 http://www.ncku.edu.tw/~iaalab/sepeo/Knowledge/vehicle/rocket/rocket.htm

二維彈性碰撞 假如兩物體相互碰撞且其運動並未沿著單一軸(碰撞並非正面的)，此碰撞爲二維的；若兩物體系統爲封閉且孤立的，動量守恆定律適用於此碰撞，並可寫爲

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}, \qquad (9\text{-}77)$$

以分量型式表示，守恆定律給出兩個描述碰撞之式(各對應二維度之一維)

$p_{1i,x} + p_{2i,x} = p_{1f,x} + p_{2f,x}$ and $p_{1i,y} + p_{2i,y} = p_{1f,y} + p_{2f,y}$.

假如碰撞亦爲**彈性的**，由動能守恆得到第三個方程式

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}. \qquad (9\text{-}78)$$

變質量系統 火箭在無外力作用下，其瞬時加速度由下式給定

$$R\,v_{rel} = M a, \quad (\text{第一火箭式}) \quad (9\text{-}87)$$

其中 M 爲火箭的瞬時質量(包含未耗盡的燃料)，R (= $-dM/dt$)爲燃料的消耗速率與 v_{rel} 爲燃料相對於火箭的噴出速率；$R v_{rel}$ 項爲火箭引擎的**推力**。對固定 R 與 v_{rel} 的火箭，當其質量從 M_i 變化至 M_f，其速度如下所示從 v_i 變化至 v_f

$$v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} \quad (\text{第二火箭式}). \quad (9\text{-}88)$$

(積分公式$\int dx/x = \ln x$)

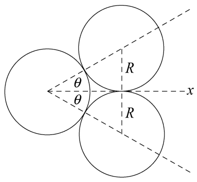
Prob. 10-1, 2, 4, 10, 15, 21, 26, 31, 39, 41, 47, 51, 55, 63, 67, 78, 83, 84, 124, 125 (tentatively)

130.• The diagram below shows the situation as the incident ball (the left-most ball) makes contact with the other two. It exerts an impulse of the same magnitude on each ball, along the line that joins the centers of the incident ball and the target ball. The target balls leave the collision along those lines, while the incident ball leaves the collision along the x axis. The three dotted lines that join the centers of the balls in contact form an equilateral triangle, so both of the angles marked θ are 30°. Let v_0 be the velocity of the incident ball before the collision and V be its velocity afterward. The two target balls leave the collision with the same speed. Let v represent that speed. Each ball has mass m. Since the x component of the total momentum of the three-ball system is conserved,

$$mv_0 = mV + 2mv\cos\theta,$$

and since the total kinetic energy is conserved,

$$(½)mv_0^2 = (½)mV^2 + 2(½)mv^2.$$



We know the directions in which the target balls leave the collision so we first eliminate V and solve for v. The momentum eq. gives $V = v_0 - 2v\cos\theta$, so

$$V^2 = v_0^2 - 4\,v_0 v\cos\theta + 4v^2\cos^2\theta,$$

and the energy eq. becomes $v_0^2 = v_0^2 - 4v_0 v\cos\theta + 4\,v^2\cos^2\theta + 2v^2$. Therefore,

$$v = \frac{2\cos\theta}{1+2\cos^2\theta} v_0 = \frac{2\cos 30°}{1+2\cos^2 30°}(10) = 6.93 \text{ (m/s)}.$$

(**a**) The discussion and computation above determines the final speed of ball 2 (as labeled in Fig. 9-83) to be 6.9 m/s. (**b**) The direction of ball 2 is at 30° counterclockwise from the $+x$ axis. (**c**) Similarly, the final speed of ball 3 is 6.9 m/s. (**d**) The direction of ball 3 is at –30° counterclockwise from the $+x$ axis. (**e**) Now we use the momentum eq. to find the final velocity of ball 1:

$$V = v_0 - 2v\cos\theta = 10 \text{ m/s} - 2(6.93 \text{ m/s})\cos 30° = -2.0 \text{ m/s}.$$

So the speed of ball 1 is $|V|$ = 2.0 m/s. (**f**) The minus sign indicates that it bounces back in the $-x$ direction. The angle is –180°. (參考題)