Chapter **8** *Potential Energy and Conservation of Energy*

Note TE = thermal energy, KE = kinetic energy, PE = potential energy, ME = mechanical energy, EPE = elastic potential energy, GPE = gravitational potential energy.

03. (**a**) The force of gravity is constant, so the work it does is given by $W = \boldsymbol{F}\cdot\boldsymbol{d}$, where $\boldsymbol{F}$ is the force and $\boldsymbol{d}$ is the displacement. The force is vertically downward and has magnitude mg, where m is the mass of the flake, so this reduces to $W_g = mgh$, where h is the height from which the flake falls. This is equal to the radius r of the bowl. Thus

$$W_g = mgr = (2.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(22.0\times10^{-2}\text{ m}) = 4.31\times10^{-3}\text{ J}.$$

(**b**) The force of gravity is conservative, so the change in GPE of the flake-Earth system is the negative of the work done: $\Delta U_g = -W_g = -4.31\times10^{-3}$ J. (**c**) The PE when the flake is at the top is greater than when it is at the bottom by $|\Delta U_g|$. If $U_g = 0$ at the bottom, then $U_g = +4.31\times10^{-3}$ J at the top. (**d**) If $U_g = 0$ at the top, then $U_g = -4.31\times10^{-3}$ J at the bottom. (**e**) All the answers are proportional to the mass of the flake. If the mass is doubled, all answers are doubled.

04. (**a**) The only force that does work on the ball is the force of gravity; the force of the rod is perpendicular to the path of the ball and so does no work. In going from its initial position to the lowest point on its path, the ball moves vertically through a distance equal to the length L of the rod, so the work done by the force of gravity is

$$W_g = mgL = (0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = 1.51\text{ J}.$$

(**b**) In going from its initial position to the highest point on its path, the ball moves vertically through a distance equal to L, but this time the displacement is upward, opposite the direction of the force of gravity. The work done by the force of gravity is

$$W_g = -mgL = -(0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = -1.51\text{ J}.$$

(**c**) The final position of the ball is at the same height as its initial position. The displacement is horizontal, perpendicular to the force of gravity. The force of gravity does no work during this displacement. (**d**) The force of gravity is conservative. The change in the GPE of the ball-Earth system is the negative of the work done by gravity:

$$\Delta U_g = -mgL = -(0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = -1.51\text{ J}.$$

as the ball goes to the lowest point. (**e**) Continuing this line of reasoning, we find

$$\Delta U_g = +mgL = -(0.341\text{ kg})(9.80\text{ m/s}^2)(0.452\text{ m}) = 1.51\text{ J}.$$

as it goes to the highest point. (**f**) Continuing this line of reasoning, we have $\Delta U_g = 0$ as it goes to the point at the same height. (**g**) The change in the GPE depends only on the initial and final positions of the ball, not on its speed anywhere. The change in the PE is the *same* since the initial and final positions are the same.

08. The main challenge for students in this type of problem seems to be working out the trigonometry in order to obtain the height of the ball (relative to the low point of the swing) $h = L - L\cos\theta$ (for angle θ measured from vertical as shown in Fig. 8-29). Once this relation (which we will not derive here since we have found this to be most easily illustrated at the blackboard) is established, then the principal results of this problem follow from Eq. 7-12 (for W_g) and Eq. 8-9 (for U). (**a**) The vertical component of the displacement vector is downward with magnitude h, so we obtain

$$W = \vec{F}_g\cdot\vec{d} = mgh = mgL(1-\cos\theta) = (5.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})(1-\cos30°) = 13.1\text{ J}.$$

(**b**) From Eq. 8-1, we have $\Delta U_g = -W_g = -mgL(1-\cos\theta) = -13.1$ J. (**c**) With $y = h$, Eq. 8-9 yields $U_g = mgL(1-\cos\theta) = 13.1$ J. (**d**) As the angle increases, we intuitively see that the height h increases (and, less obviously, from the mathematics, we see that $\cos\theta$ decreases so that $1-\cos\theta$ increases), so the answers to parts (a) and (c) increase, and the absolute value of the answer to part (b) also increases.

12. We use Eq. 8-18, representing the conservation of ME (which neglects friction and other dissipative effects). (**a**) In the solution to Pb. 4 we found $\Delta U = mgL$ as it goes to the highest point. Thus, we have

$$\Delta K + \Delta U_g = 0 \quad\text{or}\quad K_{\text{top}} - K_0 + mgL = 0,$$

which, upon requiring $K_{\text{top}} = 0$, gives $K_0 = mgL$ and thus leads to

$$v_0 = \sqrt{\frac{2K_0}{m}} = \sqrt{2gL} = \sqrt{2(9.80\text{ m/s}^2)(0.452\text{ m})} = 2.98\text{ m/s}.$$

(**b**) We also found in the Pb.4 that the PE change is $\Delta U_g = -mgL$ in going from the initial point to the lowest point (the bottom). Thus,

$$\Delta K + \Delta U_g = 0 \quad\text{or}\quad K_{\text{bottom}} - K_0 - mgL = 0,$$

which, with $K_0 = mgL$, leads to $K_{\text{bottom}} = 2mgL$. Therefore,

$$v_{\text{bottom}} = \sqrt{2K_{\text{bottom}}/m} = \sqrt{4gL} = \sqrt{4(9.80\text{ m/s}^2)(0.452\text{ m})} = 4.21\text{ m/s}.$$

(**c**) Since there is no change in height (going from initial point to the rightmost point), then $\Delta U_g = 0$, which implies $\Delta K = 0$. As a result, the speed is the same as what it was initially,

$$v_{\text{right}} = v_0 = 2.98 \text{ m/s}.$$

(**d**) It is evident from the above manipulations that the results do not depend on mass. Thus, a different mass for the ball must lead to the same results.

18. We denote m as the mass of the block, $h = 0.40$ m as the height from which it dropped (measured from the relaxed position of the spring), and x the compression of the spring (measured downward so that it yields a positive value). Our reference point for the GPE is the initial position of the block. The block drops a total distance $h+x$, and the final GPE is $-mg(h+x)$. The spring PE is $(½)kx^2$ in the final situation, and the KE is zero both at the beginning and end. Since energy is conserved

$$K_i + U_i = K_f + U_f \text{ or } 0 = -mg(h+x) + (½)kx^2,$$

which is a second degree eq. in x. Using the quadratic formula, its solution is

$$x = \frac{mg \pm \sqrt{(mg)^2 + 2mgh\,k}}{k}.$$

Now $mg = 19.6$ N, $h = 0.40$ m, and $k = 1960$ N/m, and we choose the positive root so that $x > 0$,

$$x = \frac{19.6 \pm \sqrt{19.6^2 + 2(19.6)(0.40)(1960)}}{1960} = 0.10 \text{ (m)}.$$

21. (**a**) At Q the block (which is in circular motion at that point) experiences a centripetal acceleration v^2/R leftward. We find v^2 from energy conservation:

$$K_p + U_p = K_Q + U_Q \text{ or } 0 + mgL = (½)mv^2 + mgR.$$

Using the fact that $h = 5R$, we find $mv^2 = 8mgR$. Thus, the horizontal component of the net force on the block at Q is $F = mv^2/R = 8mg = 8(0.032 \text{ kg})(9.8 \text{ m/s}^2) = 2.5$ N, and points left (in the same direction as $\boldsymbol{a}$). (**b**) The downward component of the net force on the block at Q is the downward force of gravity $F = mg = (0.032 \text{ kg})(9.8 \text{ m/s}^2) = 0.31$ N. (**c**) To barely make the top of the loop, the centripetal force there must equal the force of gravity:

$$m\frac{v_{top}^2}{R} = mg \Rightarrow \; mv_{top}^2 = mgR.$$

This requires a different value of h than was used above.

$$K_p + U_p = K_{top} + U_{top},\; 0 + mgh = (½)mv_{top}^2 + mgh_{top},$$
$$mgh = (½)(mgR) + mg(2R).$$

Consequently, $h = 2.5R = (2.5)(0.12 \text{ m}) = 0.3$ m. (**d**) The normal force F_N, for speeds v_t greater than $(gR)^{1/2}$ (which are the only possibilities for non-zero F_N — see the solution in the previous part), obeys

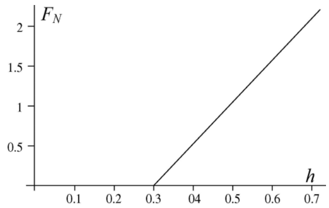
$$F_N = m\frac{v_t^2}{R} - mg,$$

from Newton's second law. Since v_t^2 is related to h by energy conservation

$$K_p + U_p = K_{top} + U_{top} \quad \Rightarrow \quad gh = \frac{1}{2}v_t^2 + 2gR,$$

then the normal force, as a function for h (so long as $h \geq 2.5R$ — see solution in previous part), becomes

$$F_N = 2mg\frac{h}{R} - 5mg.$$



Thus, the graph for $h \geq 2.5R$ consists of a straight line of positive slope $2mg/R$ (which can be set to some convenient values for graphing purposes). Note that for $h \leq 2.5R$, the normal force is zero.

23. (**a**) As the string reaches its lowest point, its original PE $U = mgL$ (measured relative to the lowest point) is converted into KE. Thus,

$$mgL = (½)mv^2 \quad \Rightarrow \quad v = \sqrt{2gL}.$$

With $L = 1.20$ m we obtain $v = 4.85$ m/s. (**b**) In this case, the total ME is shared between kinetic $(½)mv_b^2$ and potential mgy_b. We note that $y_b = 2r$ where $r = L-d = 0.450$ m. Energy conservation leads to $mgL = (½)mv_b^2 + mgy_b$, which yields $v_b = \sqrt{2gL - 2g(2r)} = 2.42$ m/s.

28. We convert to SI units and choose upward as the $+y$ direction. Also, the relaxed position of the top end of the spring is the origin, so the initial compression of the spring (defining an equilibrium situation between the spring force and the force of gravity) is $y_0 = -0.100$ m and the additional compression brings it to the position $y_1 = -0.400$ m. (**a**) When the stone is in the equilibrium ($a = 0$) position, Newton's second law becomes

$$\vec{F}_{net} = m\vec{g},\; F_{\text{spring}} - mg = 0,$$
$$-k(-0.100) - (8.00)(9.80) = 0,$$

where Hooke's law (Eq. 7-21) has been used. This leads to a spring constant equal to $k = 784$ N/m.

(**b**) With the additional compression (and release) the acceleration is no longer zero, and the stone will start moving upwards, turning some of its EPE (stored in the spring) into KE. The amount of EPE at the moment of release is, using Eq. 8-11,

$$U = (½)ky_1^2 = (½)(784)(-0.400)^2 = 62.7 \text{ (J)}.$$

(**c**) Its maximum height y_2 is beyond the point that the stone separates from the spring (entering free-fall motion). As usual, it is characterized by having (momentarily) zero speed. If we choose the y_1 position as the reference position in computing the GPE, then

$$K_1 + U_1 = K_2 + U_2 \quad \text{or} \quad 0 + (½)ky_1^2 = 0 + mgh,$$

where $h = y_2 - y_1$ is the height above the release point. Thus, mgh (the GPE) is seen to be equal to the previous answer, 62.7 J, and we proceed with the solution in the next part.

(**d**) We find $h = ky_1^2/(2mg) = 0.800$ m, or 80.0 cm.

29. We refer to its starting point as A, the point where it first comes into contact with the spring as B, and the point where the spring is compressed $|x| = 0.055$ m as C. Point C is our reference point for computing GPE. EPE (of the spring) is zero when the spring is relaxed. Information given in the second sentence allows us to compute the spring constant. From Hooke's law, we find

$$k = F / x = 270 \text{ N} / 0.02 \text{ m} = 1.35\times10^4 \text{ N/m}.$$

(**a**) The distance between points A and B is ℓ and we note that the total sliding distance $\ell+x$ is related to the initial height h of the block (measured relative to C) by $h = (\ell + |x|)\sin\theta$,
where the incline angle θ is 30°. ME conservation leads to

$$K_A + U_A = K_C + U_C \text{ or } 0 + mgh = 0 + (½)kx^2,$$

which yields

$$h = kx^2/(2mg) = (1.35\times10^4 \text{ N/m})(0.055 \text{ m})^2/ [2(12 \text{ kg})(9.80 \text{ m/s}^2)] = 0.174 \text{ m}.$$

Therefore,

$$\ell + |x| = h / \sin\theta = 0.174 \text{ m} / \sin30° = 0.35 \text{ m}.$$

(**b**) From this result, we find $\ell = 0.35\text{m} - 0.055\text{m} = 0.29$ m, which means that $\Delta y = -\ell\sin\theta = -0.15$ m in sliding from point A to point B. Thus, Eq. 8-18 gives

$$\Delta K + \Delta U = 0 \quad \text{or} \quad (½)mv_B^2 + mg\Delta h = 0,$$

which yields $v_B = \sqrt{-2g\Delta h} = \sqrt{-2(9.80)(-0.15)} = 1.7$ (m/s).

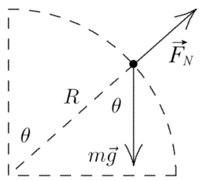
34. The distance the marble travels is determined by its initial speed (and the methods of Chapter 4), and the initial speed is determined (using energy conservation) by the original compression of the spring. We denote h as the height of the table, and x as the horizontal distance to the point where the marble lands. Then $x = v_0 t$ and $h = (½)gt^2$ (since the vertical component of the marble's "launch velocity" is zero). From these we find $x = v_0 (2h/g)^{1/2}$. We note from this that the distance to the landing point is directly proportional to the initial speed. We denote $v_{0,1}$ be the initial speed of the first shot and $D_1 = 2.20-0.27 = 1.93$ m be the horizontal distance to its landing point; similarly, $v_{0,2}$ is the initial speed of the second shot and $D = 2.20$ m is the horizontal distance to its landing spot. Then

$$v_{0,2}/v_{0,1} = D/D_1 \quad\Rightarrow\quad v_{0,2} = (D/D_1)\, v_{0,1}\,.$$

When the spring is compressed an amount ℓ, the EPE is $(½)k\ell^2$. When the marble leaves the spring its KE is $(½)mv_0^2$. ME is conserved: $(½)mv_0^2 = (½)k\ell^2$, and we see that the initial speed of the marble is directly proportional to the original compression of the spring. If ℓ_1 is the compression for the first shot and ℓ_2 is the compression for the second, then $v_{0,2} = (\ell_2/\ell_1)v_{0,1}$. Relating this to the previous result, we obtain

$$\ell_2 = (D/D_1)\,\ell_1 = (2.20 \text{ m}/1.93 \text{ m})(1.10 \text{ cm}) = 1.25 \text{ cm}.$$

36. The free-body diagram for the boy is shown below. $\boldsymbol{F}_N$ is the normal force of the ice on him and m is his mass. The net inward force is $mg\cos\theta - F_N$ and, according to Newton's second law, this must be equal to mv^2/R, where v is the speed of the boy. At the point where the boy leaves the ice $F_N = 0$, so $g\cos\theta = v^2/R$. We wish to find his speed. If the GPE is taken to be zero when he is at the top of the ice mound, then his PE at the time shown is



$$U = -mgR\,(1-\cos\theta).$$

He starts from rest and his kinetic energy at the time shown is $(½)mv^2$. Thus conservation of energy gives

$$0 = (½)mv^2 - mgR\,(1-\cos\theta),$$

or $v^2 = 2gR(1-\cos\theta)$. We substitute this expression into the eq. developed from the second law to obtain $g\cos\theta = 2g(1-\cos\theta)$. This gives $\cos\theta = 2/3$. The height of the boy above the bottom of the mound is

$$h = R\cos\theta = (2/3)R = (2/3)(13.8 \text{ m}) = 9.20 \text{ m}.$$

40. (**a**) Using Eq. 7-8, we have $W_{\text{applied}} = (8.0 \text{ N})(0.70 \text{ m}) = 5.6$ J. (**b**) Using Eq. 8-31, the TE generated is

$$\Delta E_{\text{th}} = f_k d = (5.0 \text{ N})(0.70 \text{ m}) = 3.5 \text{ J}.$$

59. The initial and final KEs are zero, and we set up energy conservation in the form of Eq. 8-33 (with $W = 0$) according to our assumptions. Certainly, it can only come to a permanent stop somewhere in the flat part, but the question is whether this occurs during its first pass through (going rightward) or its second pass through (going leftward) or its third pass through (going rightward again), and so on. If it occurs during its first pass through, then the TE generated is $\Delta E_{th} = f_k d$ where $d \le L$ and $f_k = \mu_k mg$. If it occurs during its second pass through, then the total TE is $\Delta E_{th} = \mu_k mg(L+d)$ where we again use the symbol d for how far through the level area it goes during that last pass (so $0 \le d \le L$). Generalizing to the n^{th} pass through, we see that

$$\Delta E_{\text{th}} = \mu_k mg\,[(n-1)L + d].$$

In this way, we have

$$mgh = \mu_k mg\,[\,(n-1)L + d\,]$$

which simplifies (when $h = L/2$ is inserted) to

$$\frac{d}{L} = 1 + \frac{1}{2\mu_k} - n\,.$$

The first two terms give $1+1/(2\mu_k) = 3.5$ so that the requirement $0 \le d/L \le 1$ demands that $n = 3$. We arrive at the conclusion that $d/L = 1/2$, or

$$d = L/2 = (40 \text{ cm})/2 = 20 \text{ cm}$$

and that this occurs on its third pass through the flat region.

97. Eq. 8-33 gives

$mgy_f = K_i + mgy_i - E_{th}$, (0.50)(9.8)(0.80) = (½)(0.50)(4.00)^2 + (0.50)(9.8)(0) – E_{th}

which yields E_{th} = 4.00 – 3.92 = 0.080 (J).

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

Ex.1: Prob.8-51; Ex.2: Prob.8-129.

重點整理－第 8 章 位能與能量守恆

那推理有何錯誤？

保守力 若一力對沿著封閉路徑(從起點再回到起點)運動的質點所作的淨功為零，則其為**保守力**。等同地，若一力對於兩點間運動的質點所作的淨功與質點運動的路徑無關時，亦為保守力。**重力和彈力為保守力；動摩擦力為非保守力**。

位能 位能是與保守力作用的系統之狀態有關的能量，當保守力對系統內的質點作功時，系統的位能改變量ΔU為

$$\Delta U = -W. \qquad (8\text{-}1)$$

若質點從點 x_i 運動到點 x_f，系統位能改變量為

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx. \qquad (8\text{-}6)$$

重力位能 與地球及其附近的質點所組成的系統有關的位能為**重力位能**。假如質點從高度 y_i 運動到高度 y_f 時，質點–地球系統的重力位能改變量為 $\Delta U = mg(y_f - y_i) = mg\Delta y.$ (8-7)

假如質點的參考點設為 $y_i = 0$ 及對應的系統重力位能設為 $U_i = 0$ 時，則當質點於任意高度 y 的重力位能 U 為 $U(y) = mgy$. (8-9)

彈性位能 **彈性位能**是與彈性體之壓縮或伸張狀態有關的能量。對於施加彈力 $F = -kx$ 之彈簧，當其自由端有一位移 x 時，**彈性位能**為

$$U(x) = \frac{1}{2}kx^2 \qquad (8\text{-}11)$$

參考組態為彈簧在其鬆弛長度，即 $x = 0$ 及 $U = 0$。

力學能 一系統的**力學能** E_{mec} 為其動能 K 和位能 U 之和： $E_{mec} = K + U$, (8-12)

一孤立系統為一無外力造成能量改變的系統。假如在一孤立系統內只有保守力作功，則系統力學能 E_{mec} 不會改變。**力學能守恆原理**寫為

$$K_2 + U_2 = K_1 + U_1 , \qquad (8\text{-}17)$$

其中下標代表於能量移轉過程中不同的瞬間。力學能守恆定律亦可寫成

$$\Delta E_{mec} = \Delta K + \Delta U = 0. \qquad (8\text{-}18)$$

位能曲線 若對質點受一維力 $F(x)$作用之系統，其位能函數 $U(x)$已知，則其作用力為

$$F(x) = -dU/dx. \qquad (8\text{-}22)$$

若將 $U(x)$劃於圖上，於任一位置 x，力 $F(x)$為該處曲線**切線斜率的負值**，而質點動能為

$$K(x) = E_{mec} - U(x), \qquad (8\text{-}24)$$

式中 E_{mec} 為系統的力學能，**折返點**為質點運動方向改變之點(該處 K = 0 或 v = 0)。質點在 $U(x)$曲線的斜率為 0 (該處 $F(x)$ = 0)之點，其處於**平衡**。

外力對系統所作的功 功 W 為藉著作用於系統之外力而從系統移轉出或移轉入的能量。當有一個以上的外力作用於系統時，其**淨功**即為移轉的能量。當無摩擦力時，對系統所作的功與系統力學能的改變量ΔE_{mec}相等，

$$W = \Delta E_{mec} = \Delta K + \Delta U. \qquad (8\text{-}26,25)$$

當系統內有動摩擦力作用時，則系統的熱能ΔE_{th}會改變(此能量與系統內原子及分子的隨機運動有關)。對系統作的功為

$$W = \Delta E_{mec} + \Delta E_{th}. \qquad (8\text{-}33)$$

熱能改變量ΔE_{th} 與動摩擦力大小 f_k 及外力造成的位移量 d 有關

$$\Delta E_{th} = f_k d. \qquad (8\text{-}31)$$

能量守恆 系統的總能量 E (其力學能、內能及熱能之和)只依移轉入系統或從系統移轉出的能量而改變。這實驗的事實即為**能量守恆定律**。假如對系統作功 W，則

$$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}. \qquad (8\text{-}35)$$

假如系統為孤立的(W = 0)，

$$\Delta E_{mec} + \Delta E_{th} + \Delta E_{int} = 0, \qquad (8\text{-}36)$$

及 $$E_{mec,2} = E_{mec,1} - \Delta E_{th} - \Delta E_{int} , \qquad (8\text{-}37)$$

其中下標 1 和 2 代表兩個不同瞬間。

功率 由作用力產生的功率為力移轉能量的時變率。假如在一段時間Δt 內，力造成的能量移轉量為ΔE，則該力的**平均功率**為

$$P_{av} = \Delta E/\Delta t, \qquad (8\text{-}40)$$

由力產生的**瞬時功率**為

$$P = dE/dt. \qquad (8\text{-}41)$$

64.• We use conservation of ME: the ME must be the same at the top of the swing as it is initially. Newton's second law is used to find the speed, and hence the KE, at the top. There the tension force T of the string and the force of gravity are both downward, toward the center of the circle. We notice that the radius of the circle is $r = L - d$, so the law can be written

$$T + mg = m\frac{v^2}{L-d},$$

where v is the speed and m is the mass of the ball. When the ball passes the highest point with the least possible speed, the tension is zero. Then

$$mg = m\frac{v^2}{L-d} \Rightarrow v = \sqrt{g(L-d)}.$$

We take the GPE of the ball-Earth system to be zero when the ball is at the bottom of its swing. Then the initial PE is mgL. The initial KE is zero since the ball starts from rest. The final PE, at the top of the swing, is $2mg(L-d)$ and the final KE is $(½)mv^2 = mg(L-d)/2$ using the above result for v. Conservation of energy yields

$$mgL = 2mg(L-d) + (½)mg(L-d) \Rightarrow d = (3/5)L.$$

With $L = 1.20$ m, we have $d = 0.60(1.20\text{ m}) = 0.72$ m. Notice that if d is greater than this value, so the highest point is lower, then the speed of the ball is greater as it reaches that point and the ball passes the point. If d is less, the ball cannot go around. Thus the value we found for d is a lower limit.

68.•* We use SI units so $m = 0.030$ kg and $d = 0.12$ m. (**a**) Since there is no change in height (and we assume no changes in EPE), then $\Delta U = 0$ and we have

$$\Delta E_{mech} = \Delta K = -(½)mv_0^2 = -3.8\times10^3 \text{ J}.$$

where $v_0 = 500$ m/s and the final speed is zero. (**b**) By Eq. 8-33 (with $W = 0$) we have $\Delta E_{th} = 3.8\times10^3$ J, which implies

$$f = \Delta E_{th} / d = 3.1\times10^4 \text{ N},$$

using Eq. 8-31 with f_k replaced by f (effectively generalizing that eq. to include a greater variety of dissipative forces than just those obeying Eq. 6-2).

Pb. 9-1, 3, 5, 15, 17, 20, 26, 29, 33, 44, 46, 50, 53, 54, 56, 60, 66, 71, 77, 81, 109, 130, 131, 133, 135 (tentatively)

47.* We work this using the English units (with g = 32 ft/s), but for consistency we convert the weight to pounds $mg = (9.0\text{oz})(1\text{lb}/16\text{oz}) = 0.56\text{lb}$, which implies $m = 0.018$ lb·s²/ft (which can be phrased as 0.018 slug as explained in Appendix D). And we convert the initial speed to feet-per-second

$$v_i = (81.8 \text{ mi/h})[(5280 \text{ ft/mi})/3600 \text{ s/h}] = 120 \text{ ft/s}$$

or a more "direct" conversion from Appendix D can be used. Equation 8-30 provides $\Delta E_{th} = -\Delta E_{mec}$ for the energy "lost" in the sense of this problem. Thus,

$$\Delta E_{th} = (½)m(v_i^2 - v_f^2) = (½)(0.018)(120^2 - 110) = 20 \text{ (ft·lb)}.$$

70.•* The work required is the change in the GPE as a result of the chain being pulled onto the table. Dividing the hanging chain into a large number of infinitesimal segments, each of length dy, we note that the mass of a segment is $(m/L)dy$ and the change in PE of a segment when it is a distance $|y|$ below the table top is

$$dU = (m/L)\, g\, |y|\, dy = -(m/L)\, g\, y\, dy.$$

Since y is negative-valued (we have $+y$ upward and the origin is at the tabletop). The total PE change is

$$\Delta U = -\frac{mg}{L}\int_{-L/4}^{0} y\,dy = \frac{1}{2}\frac{mg}{L}\left(\frac{L}{4}\right)^2 = \frac{1}{32}mgL.$$

The work required to pull the chain onto the table is therefore $W = \Delta U = mgL/32$

$$= (0.012 \text{ kg})(9.8 \text{ m/s}^2)(0.28 \text{ m})/32 = 0.0010 \text{ J}.$$

law 定律; principle 原理; theorem 定理; work 功; power 功率; external force 外力; nonconservative/conservative force 非保守/保守力; gravitational/elastic potential energy 重力/彈性位能; thermal energy 熱能; mechanical energy 力學能; conservation of mechanical energy 力學能守恆; law of conservation of energy 能量守恆定律; reference configuration/point 參考組態/點; stable/neutral equilibrium 穩定/隨遇平衡; bob 懸錘; Easter 復活節; beagle 小獵犬; limb 大樹枝; pendulum 擺; round trip 來回旅行;rock-climbing 攀岩; sloth 樹獺; tamale 墨西哥烹調; turning point 折返點;

•**挑戰題**•若考慮空氣阻力，試證明拋體下降時間將大於上升時間。

•**備忘錄**•