Chapter 4 *Motion in Two and Three Dimensions*

機車跳躍者如何決定其必需的起飛速率？

04. We choose a coordinate system with origin at the clock center and $+x$ rightward (towards the "3:00" position) and $+y$ upward (towards "12:00"). (**a**) In unit-vector notation, we have (in cm) $\mathbf{r}_1 = 10\ \mathbf{i}$ cm and $\mathbf{r}_2 = -10\ \mathbf{j}$cm. Thus, Eq. 4-2 gives $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = (-10\ \mathbf{i} - 10\ \mathbf{j})$cm. Thus, the magnitude is given by $|\Delta\mathbf{r}| = [(-10)^2+(-10)^2]^{1/2} = 14$ (cm). (**b**) The angle is

$$\theta = \tan(-10/-10) = 45° \text{ or } -135°.$$

We choose $-135°$ since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = -10\mathbf{i} - 10\mathbf{j} \rightarrow (14\angle{-135°})$. (**c**) In this case, $\mathbf{r}_1 = -10\,\mathbf{j}$cm and $\mathbf{r}_2 = 10\,\mathbf{j}$cm, and $\Delta\mathbf{r} = 20\,\mathbf{j}$cm. Thus, $|\Delta\vec{r}| = 20$ cm. (**d**) The angle is given by

$$\theta = \tan(20/0) = 90°.$$

(**e**) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero. (**f**) The corresponding angle for a full-hour sweep is also zero.

07. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{av} = \frac{(-2.0\,\hat{\text{i}} + 8.0\,\hat{\text{j}} - 2.0\,\hat{\text{k}}) - (5.0\,\hat{\text{i}} - 6.0\,\hat{\text{j}} + 2.0\,\hat{\text{k}})}{10}$$

$$= (-0.7\ \hat{\text{i}} + 1.40\,\hat{\text{j}} - 0.40\ \hat{\text{k}}) \text{ m/s}.$$

09. We apply Eq. 4-10 and Eq. 4-16. (**a**) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = (d/dt)(\hat{\text{i}} + 4t^2\,\hat{\text{j}} + t\,\hat{\text{k}}) = 8t\,\hat{\text{j}} + \hat{\text{k}}.$$

(**b**) Taking another derivative with respect to time leads to, in SI units (m/s^2),

$$\vec{a} = (d/dt)(8t\,\hat{\text{j}} + \hat{\text{k}}) = 8\,\hat{\text{j}}.$$

18. We use Eq. 4-26

$$R_{max} = (v_0^2/g)\sin 2\theta_0 = v_0^2/g = 9.5^2/9.80$$
$$= 9.209 \approx 9.21 \text{ (m)}.$$

to compare with Powell's long jump; the difference from R_{max} is only $R = (9.21-8.95) = 0.259$ (m).

20. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. (**a**) With the origin at the initial point (edge of table), the y coordinate of the ball is given by $y = -(½)gt^2$. If t is the time of flight and $y = -1.20$ m indicates the level at which the ball hits the floor, then

$$t = \sqrt{2(-1.20)/(-9.80)} = 0.495 \text{ (s)}.$$

(**b**) The initial (horizontal) velocity of the ball is $\mathbf{v} = v_0\mathbf{i}$. Since $x = 1.52$m is the horizontal position of its impact point with the floor, we have $x = v_0 t$. Thus,

$$v_0 = x/t = 1.52 / 0.495 = 3.07 \text{ (m/s)}.$$

29. At maximum height, we observe $v_y = 0$ and denote $v_x = v$ (which is also equal to v_{0x}). In this notation, we have $v_0 = 5v$. Next, we observe $v_0\cos\theta_0 = v_{0x} = v$, so that we arrive at an equation (where $v \neq 0$ cancels) which can be solved for θ_0:

$$(5v)\cos\theta_0 = v \Rightarrow \theta_0 = \cos^{-1}(1/5) = 78.5°.$$

31. The coordinate origin is taken at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let θ_0 be the firing angle. If the target is a distance d away, then its coordinates are $x = d$, $y = 0$. The projectile motion equations lead to $d = v_0 t\cos\theta_0$ and $0 = v_0 t\sin\theta_0 - (½)gt^2$. Eliminating t leads to $2v_0^2\sin\theta_0\cos\theta_0 - gd = 0$. Using $\sin\theta_0\cos\theta_0 = (½)\sin(2\theta_0)$, we obtain

$$v_0^2\sin(2\theta_0) = gd \Rightarrow \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80)(45.7)}{(460)(460)},$$

which yields $\sin(2\theta_0) = 2.11\times10^{-3}$ and consequently $\theta_0 = 0.0606°$. If the gun is aimed at a point a distance ℓ above the target, then $\tan\theta_0 = \ell/d$ so that

$$\ell = d\tan\theta_0 = (45.7 \text{ m})\tan(0.0606°)$$
$$= 0.0484 \text{ m} = 4.84 \text{ cm}.$$

***Solution* 2**. Owing to the equal height of firing point and target, the falling height h from the pointed spot is $h = (½)gt^2$ with t being the flight time.

We have $t = 45.7/460 = 9.934\times10^{-2}$ (s)

and $h = (½)gt^2 = 0.0484$ (m).

32. The initial velocity is horizontal so that $v_{0y} = 0$ and $v_{0x} = v_0 = 161$ km/h $= 44.72$ m/s. (**a**) With the origin at the initial point (where the ball leaves the pitcher's hand), the y coordinate of the ball is given by $y = -(½)gt^2$, and the x coordinate is given by $x = v_0 t$. From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if $x = 18.3/2$ m, then $t = (18.3/2)/44.7 = 0.205$ (s). (**b**) And the time to travel the next 18.3/2 m must also be 0.205 s. It can be useful to write the horizontal equation as $x = v_0 t$ in order that this result can be seen more clearly. (**c**) From $y = -(½)gt^2$, we see that the ball has reached the height of $(½)(9.80)(0.205)^2 = 0.205$ (m) at the moment the ball is halfway to the batter. (**d**) The ball's height when it reaches the batter is $-(½)(9.80)(0.409)^2 = -0.820$(m), which, when subtracted from the previous result, implies it has fallen another 0.615 m. Since the value of y is not simply proportional to t, we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial y-velocity for the first half of the motion is not the same as the "initial" y-velocity for the second half of the motion.

37. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find v_0 directly from Eq. 4-26. (**a**) We want to know how high the ball is from the ground when it is at $x = 97.5$ m, which requires knowing the initial velocity. Using the range information and $\theta_0 = 45°$, we use Eq. 4-26 to solve for v_0:

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{9.80 \times 97.5}{1}} = 32.4\,(\text{m/s}).$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{97.5}{32.4 \times \cos 45°} = 4.26\ (\text{s}).$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin\theta_0)t - (½)gt^2 = 9.88\ \text{m},$$

which implies it does indeed clear the 7.32 m high fence. (**b**) At $t = 4.26$ s, the center of the ball is $9.88 - 7.32 = 2.56$ (m) above the fence.

44. The magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(10\ \text{m/s})^2}{25\ \text{m}} = 4.0\ \text{m/s}^2.$$

47. The magnitude of centripetal acceleration ($a = v^2/r$) and its direction (towards the center of the circle) form the basis of this problem. (**a**) If a passenger at this location experiences $\boldsymbol{a} = 1.83$ m/s^2 east, then the center of the circle is *east* of this location. And the distance is $r = v^2/a = (3.66^2)/(1.83) = 7.32$ (m). (**b**) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west. (**c**) If the direction of $\boldsymbol{a}$ experienced by the passenger is now *south*— indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

59. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle θ from the vertical is found from

$$\tan\theta = v_h/v_v = 13.9/8.0 = 1.74,$$

which yields $\theta = 60°$.

69.*AP* Since $v_y^2 = v_{0y}^2 - 2g\Delta y$, and $v_y = 0$ at the target, we obtain $v_{0y} = [2(9.80)(5.00)]^{1/2} = 9.90$ (m/s). (**a**) Since $v_0 \sin\theta_0 = v_{0y}$, with $v_0 = 12.0$ m/s, we find $\theta_0 = 55.6°$. (**b**) Now, $v_y = v_{0y} - gt$ gives $t = 9.90/9.80 = 1.01$ (s). Thus, $x = (v_0\cos\theta_0)t = 6.85$ m. (**c**) The velocity at the target has only the v_x component, which is equal to $v_{0x} = v_0\cos\theta_0 = 6.78$ m/s.

77AP. With $v_0 = 30.0$ m/s and $R = 20.0$ m, Eq. 4-26 gives $\sin 2\theta_0 = gR/v_0^2 = 0.218$. Because $\sin\theta = \sin(180° - \theta)$, there are two roots of the above eq.:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58° \text{ and } 167.4°,$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects). (**a**) The smallest angle is $\theta_0 = 6.29°$. (**b**) The greatest angle is and $\theta_0 = 83.7°$. An alternative approach to this problem in terms of Eq. 4-25 (with $y = 0$ and $\sec^2\theta = 1 + \tan^2\theta$) is possible — and leads to a quadratic equation for $\tan\theta_0$ with the roots providing these two possible θ_0 values.

85.*AP* We use a coordinate system with +x eastward and +y upward. (**a**) We note that 123° is the angle between the initial position and later position vectors, so that the angle from +x to the later position vector is 40° + 123° = 163°. In unit-vector notation, the position vectors are

$$\vec{r}_1 = 360\cos 40°\,\hat{\text{i}} + 360\sin 40°\,\hat{\text{j}} = 276\,\hat{\text{i}} + 231\,\hat{\text{j}},$$

$$\vec{r}_2 = 790\cos 163°\,\hat{\text{i}} + 790\sin 163°\,\hat{\text{j}} = -755\,\hat{\text{i}} + 231\,\hat{\text{j}},$$

respectively (in meters). Consequently, we plug into Eq. 4-3

$$\Delta\vec{r} = (-755 - 276)\,\hat{\text{i}} + (231 - 231)\,\hat{\text{j}} = -(1031\ \text{m})\,\hat{\text{i}}.$$

Thus, the magnitude of the displacement $\Delta\boldsymbol{r}$ is $|\Delta\boldsymbol{r}| = 1031$ m. (**b**) The direction of $\Delta\boldsymbol{r}$ is $-\mathbf{i}$, or westward.

88.*AP* Eq. 4-34 describes an inverse proportionality between r and a, so that a large acceleration results from a small radius. Thus, an upper limit for a corresponds to a lower limit for r. (**a**) The minimum turning radius of the train is given by

$$r_{\max} = \frac{v^2}{a_{\max}} = \frac{(216\ \text{km/h})^2}{0.50 \times 9.80\text{m/s}^2} = 7.3 \times 10^3\ \text{m}.$$

(**b**) The speed of the train must be reduced to no more than

$$v = \sqrt{ra_{\max}} = \sqrt{1000\ \text{m} \times 0.50 \times 9.80\ \text{m/s}^2} = 22\ \text{m/s}.$$

112. We apply Eq. 4-35 to solve for speed v and Eq. 4-34 to find acceleration a. (**a**) Since the radius of Earth is 6.37×10^6 m, the radius of the satellite orbit is $(6.37 \times 10^6 + 640 \times 10^3) = 7.01 \times 10^6$ (m). Therefore, the speed of the satellite is

$$v = 2\pi r / T = 2\pi(7.01 \times 10^6\ \text{m})/(98.0 \times 60\ \text{s})$$
$$= 7.49 \times 10^3\ \text{m/s}.$$

(**b**) The magnitude of the acceleration is

$$a = v^2/r = (7.49 \times 10^3)^2/(7.01 \times 10^6) = 8.00\ (\text{m/s}^2).$$

116.*AP* The radius of Earth may be found in Appendix C. (**a**) The speed of an object at Earth's equator is $v = 2\pi R/T$, where R is the radius of Earth (6.37×10^6 m) and T is the length of a day (8.64×10^4 s):

$$v = 2\pi(6.37 \times 10^6\ \text{m})/(8.64 \times 10^4\ \text{s}) = 463\ \text{m/s}.$$

The magnitude of the acceleration is given by

$$a = v^2/r = (463)^2/(6.37 \times 10^6) = 0.034\ (\text{m/s}^2).$$

(**b**) If T is the period, then $v = 2\pi R/T$ is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus, $T = 2\pi\sqrt{R/a} = 2\pi\sqrt{(6.37\times10^6)/9.80}$

$= 5.1\times10^3$ (s) = 84 (min).

118.*AP* When the escalator is stalled the speed of the person is $v_p = \ell/t$, where ℓ is the length of the escalator and t is the time the person takes to walk up it. This is v_p = (15 m)/(90 s) = 0.167 m/s. The escalator moves at v_e = (15 m)/(60 s) = 0.250 m/s. The speed of the person walking up the moving escalator is $v = v_p + v_e$ = 0.167 m/s + 0.250 m/s = 0.417 m/s and the time taken to move the length of the escalator is

$$t = \ell / v = (15\text{ m})/(0.417\text{ m/s}) = 36\text{ s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of ℓ (in meters) and the speed (in meters per second) of the person walking on the stalled escalator is $\ell/90$, the speed of the moving escalator is $\ell/60$, and the speed of the person walking on the moving escalator is $v = \ell/90+\ell/60 = \ell/36 = 0.0278\ell$. The time taken is $t = \ell/v = \ell/(0.0278\ell)$ = 36 (s) and is independent of ℓ.

132AP. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos\theta_0}\sqrt{\frac{g}{2(x\tan\theta_0 - y)}},$$

which yields v_0 = 23 ft/s for g = 32 ft/s^2, x = 13 ft, y = 3 ft, and θ_0 = 55°.

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

Ex.1, Pb. 4-22 & ***Ex.2, Pb. 4-128***.

kinematics,運動學; dynamics,動力學; mechanics,力學;parabola,拋物線; projectile motion,拋體運動; initial velocity,初速; launching,發射/下水; landing,降落/登陸; the equation of the path,路徑方程式; trajectory,彈/軌道; horizontal range,水平射程; time of flight,飛行時間; uniform circular motion,等速率圓周運動; centripetal acceleration,向心加速度, reference frame,參考(座標)系; relative motion,相對運動; ramp,坡道; high jump,跳高; shotput,推鉛球; free-throw line,罰球線; dunk shot,灌籃; pirate,海盜; sprinter 短跑選手; roller coaster,雲霄飛車; Ferris wheel,摩天輪; merry-go-round,旋轉木馬; "Top gun",捍衛戰士; dogfight,空戰;

•有關空氣阻力之效應請參閱"**牛頓打棒球**", 李靜宜譯，牛頓。

重點整理－第 4 章 二維與三維運動

•**力學**包含**運動學**與**動力學**；運動學是力學之初步，**運動學**旨在探討物體如何運動，即描述物體在空間之位置與時間的關係 $\mathbf{r} = \mathbf{r}(t)$，只用了「**空間**」及「**時間**」兩個基本概念，沒有「**力**」與「**質量**」等概念。**動力學--**探討運動之起因，物體為什麼作這樣運動。即力、以及運動物體之間的關係，研討物體運動時所遵守的定律或法則－**牛頓運動定律**。

•拋體運動：初速度(v_0, θ_0)或($v_0\cos\theta_0$, $v_0\sin\theta_0$)

水平(x)：等速 a_x = 0 & 鉛直(y)：等加速 $a_y = -g$

速度：$v_x = v_0\cos\theta_0$, $v_y = v_0\sin\theta_0 - g\,t$, ($\Delta y > 0$: up)

位置：$x - x_0 = v_0\cos\theta_0\, t$, $y - y_0 = v_0\sin\theta_0\, t - (½)gt^2$,

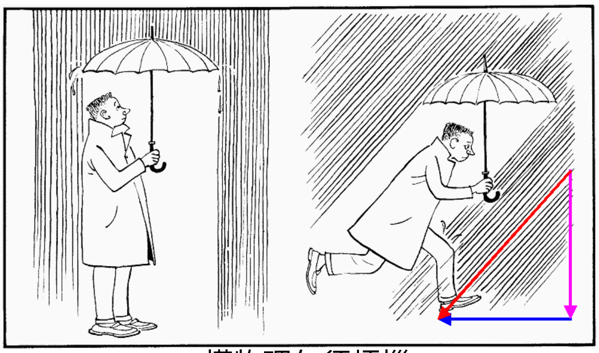
軌跡：拋物線 $y = (\tan\theta_0)x - \dfrac{g\,x^2}{2(v_0\cos\theta_0)^2}$ for $x_0 = y_0 = 0$。當出發與落地高度相等時，**水平射程**：

$$R = \frac{v_0^2}{g}\sin(2\theta_0),\ 若\ \theta_0 = 45°,\ R_{max} = \frac{v_0^2}{g}。$$

•等速率圓周運動：質點以等速率 v 於圓周(半徑 r)上運動稱之；週期 $T = 2\pi r / v$ (or $v = 2\pi r / T$), (向心)加速度方向永指向圓心，大小

$$a\ (a_c) = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2}。$$

相對運動：$\vec{r}_{PA} = \vec{r}_{PB} + \vec{r}_{BA}$，$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$



(懂物理勿須煩惱)

•**挑戰題**•某拋體於空中運動之最大爬升高度為 H，而水平射程為 R，試計算(**a**).此拋體之拋射(或出發)速度(含速率及方向)及(**b**).於空中飛行時間；假設出發高度與觸地高度相同。

•**備忘錄**