Chapter 2 *Motion Along a Straight Line*

02. Huber's speed is

$$v_0 = (200\,\text{m})/(6.509\,\text{s}) = 30.72\,\text{m/s} = 110.6\,\text{km/h},$$

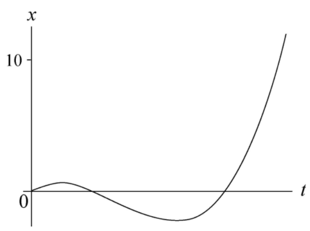
where we have used the conversion factor 1 m/s = 3.6 km/h. Since Whittingham beat Huber by 19.0 km/h, his speed is v_1 = (110.6+19.0) km/h = 129.6 km/h, or 36 m/s. Thus, the time through a distance of 200 m for Whittingham is

$$\Delta t = \Delta x\,/\,v_1 = (200\,\text{m})/(36\,\text{m/s}) = 5.554\,\text{s}.$$

05. Using $x = 3t - 4t^2 + t^3$ with SI units understood is efficient (and is the approach we will use), but if we wished to make the units explicit we would write $x = (3\text{ m/s})\,t - (4\text{ m/s}^2)\,t^2 + (1\text{ m/s}^3)\,t^3$. We will quote our answers to one or two significant figures, and not try to follow the significant figure rules rigorously. (**a**) Plugging in t = 1 s yields x = 3 – 4 + 1 = 0. (**b**) With t = 2 s we obtain $x = 3(2) - 4(2)^2 + (2)^3 = -2$ (m). (**c**) With t = 3 s we have x = 0 m. (**d**) Plugging in t = 4 s gives x = 12 m. For later reference, we also note that the position at t = 0 is x = 0. (**e**) The position at t = 0 is subtracted from the position at t = 4 s to find the displacement Δx = 12 m. (**f**) The position at t = 2 s is subtracted from the position at t = 4 s to give the displacement Δx = 14 m. Eq. 2-2, then, leads to

$$v_{av} = \Delta x\,/\,\Delta t = 14/2 = 7\text{ (m/s)}\,.$$

(**g**) The horizontal axis is $0 \le t \le 4$ with SI units understood. Not shown is a straight line drawn from the point at (t, x) = (2, –2) to the highest point shown (at t = 4 s) which would represent the answer for part (f).



09. Converting to seconds, the running times are t_1 = 147.95 s and t_2 = 148.15 s, respectively. If the runners were equally fast, then

$$S_{av1} = S_{av2} \quad\Rightarrow\quad L_1/t_1 = L_2/t_2\,.$$

From this we obtain

$$L_2 - L_1 = \left(\frac{t_2}{t_1} - 1\right)L_1 = \left(\frac{148.15}{147.95} - 1\right)L_1$$

$$= 0.00135L_1 \approx 1.4\text{ m},$$

where we set $L_1 \approx$ 1000 m in the last step. Thus, if L_1 and L_2 are no different than about 1.4 m, then runner 1 is indeed faster than runner 2. However, if L_1 is shorter than L_2 by more than 1.4 m, then runner 2 would actually be faster.

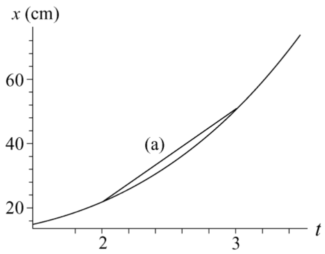
13. We use Eq. 2-2 for average velocity and Eq. 2-4 for instantaneous velocity, and work with distances in centimeters and times in seconds. (**a**) We plug into the given equation for x for t = 2.00 s and t = 3.00 s and obtain x_2 = 21.75 cm and x_3 = 50.25 cm, respectively. The average velocity during the time interval $2.00 \le t \le 3.00$ s is

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{50.25 - 21.75}{3.00 - 2.00},$$

which yields v_{av} = 28.5 cm/s. (**b**) The instantaneous velocity is $v = dx/dt = 4.5t^2$, which, at time t = 2.00 s, yields $v = (4.5)\,(2.00)^2 = 18.0$ (cm/s). (**c**) At t = 3.00 s, the instantaneous velocity is $v = (4.5)(3.00)^2 = 40.5$ (cm/s). (**d**) At t = 2.50 s, the instantaneous velocity is $v = (4.5)(2.50)^2 = 28.1$ (cm/s). (**e**) Let t_m stand for the moment when the particle is midway between x_2 and x_3 (that is, when the particle is at $x_m = (x_2+x_3)/2$ = 36 cm). Therefore,

$$x_m = 9.75 + 1.5t_m^2 \quad\Rightarrow\quad t_m = 2.596\text{ s}.$$

Thus, the instantaneous speed at this time is $v = 4.5(2.596)^2 = 30.3$ (cm/s). (**f**) The answer to part (a) is given by the slope of the straight line between t = 2 and t = 3 in this x-vs-t plot. The answers to parts (b), (c), (d) and (e) correspond to the slopes of tangent lines (not shown but easily imagined) to the curve at the appropriate points.



17. (**a**) Taking derivatives of $x(t) = 12t^2 - 2t^3$, we obtain the velocity and the acceleration functions:

$$v(t) = 24t - 6t^2 \qquad\text{and}\qquad a(t) = 24 - 12t$$

with length in meters and time in seconds. Plugging in the value t = 3 yields $x(3) = 54$ m. (**b**) Similarly, plugging in the value t = 3 yields $v(3)$ = 18 m/s. (**c**) For t = 3, $a(3) = -12$ m/s^2. (**d**) At the maximum x, we must have v = 0; eliminating the t = 0 root, the velocity equation reveals t = 24/6 = 4 (s) for the time of maximum x. Plugging t = 4 into the equation for x leads to x = 64 m for the largest x value reached by the particle. (**e**) From (d), we see that the x reaches its maximum at t = 4.0 s. (**f**) A maximum v requires a = 0, which occurs when t = 24/12 = 2.0 (s). This, inserted into the velocity equation, gives v_{max} = 24 m/s. (**g**) From (f), we see that the maximum of v occurs at t = 24/12 = 2.0 (s). (**h**) In part (e), the particle was (momentarily) motionless at t = 4 s. The acceleration at that time is readily found to be $24 - 12(4) = -24$(m/s^2). (**i**) The *average velocity* is defined by Eq. 2-2, so we see that the values of x at t = 0 and t = 3 s are needed; these are, respectively, x = 0 and x = 54 m (found in part (a)). Thus, v_{av} = (54–0)/(3–0) = 18 (m/s).

18. We use Eq. 2-2 (average velocity) and Eq. 2-7 (average acceleration). Regarding our coordinate choices, the initial position of the man is taken as the origin and his direction of motion during 5 min $\le t \le 10$ min is taken to be the positive x direction. We also use the fact that $\Delta x = v\Delta t'$ when the velocity is constant during a time interval $\Delta t'$. (**a**) The entire interval considered is $\Delta t = 8 - 2 = 6$ (min) which is equivalent to 360 s, whereas the sub-interval in which he is *moving* is only $\Delta t' = 8 - 5 = 3$ (min) = 180 (s). His position at $t = 2$ min is $x = 0$ and his position at $t = 8$ min is $x = v\Delta t' = (2.2)(180) = 396$ (m). Therefore,

$$v_{av} = (396\,\text{m} - 0)/(360\,\text{s} - 0) = 1.10\ \text{m/s}.$$

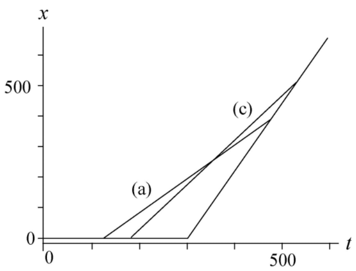
(**b**) The man is at rest at $t = 2$ min and has velocity $v = +2.2$ m/s at $t = 8$ min. Thus, keeping the answer to 3 significant figures,

$$a_{av} = (2.2\ \text{m/s} - 0)/(360\ \text{s} - 0) = 0.00611\ \text{m/s}^2 .$$

(**c**) Now, the entire interval considered is $\Delta t = 9 - 3 = 6$ min (360 s again), whereas the sub-interval in which he is moving is $\Delta t' = 9 - 5 = 4\,\text{min} = 240\ \text{s}$). His position at $t = 3$ min is $x = 0$ and his position at $t = 9$ min is $x = v\Delta t' = (2.2)(240) = 528$ (m). Therefore,

$$v_{av} = (528\,\text{m} - 0)/(360\,\text{s} - 0) = 1.47\ \text{m/s}.$$

(**d**) The man is at rest at $t = 3$ min and has velocity $v = +2.2$ m/s at $t = 9$ min. Consequently, $a_{av} = 2.2/360 = 0.00611$ (m/s^2) just as in part (b). (**e**) The horizontal line near the bottom of this x-vs-t graph represents the man standing at $x = 0$ for $0 \le t < 300$ s and the linearly rising line for $300 \le t \le 600$ s represents his constant-velocity motion. The dotted lines represent the answers to part (a) and (c) in the sense that their slopes yield those results. The graph of v-vs-t is not shown here, but would consist of two horizontal "steps" (one at $v = 0$ for $0 \le t < 300$ s and the next at $v = 2.2$ m/s for $300 \le t \le 600$ s). The indications of the average accelerations found in parts (b) and (d) would be dotted lines connecting the "steps" at the appropriate t values (the slopes of the dotted lines representing the values of a_{av}). Using the above value for θ and $h = 1.7$ m, we have $r = 5.2\times10^6$ m.



24. We separate the motion into two parts, and take the direction of motion to be positive. In part 1, the vehicle accelerates from rest to its highest speed; we are given $v_0 = 0$; $v = 20$ m/s and $a = 2.0$ m/s^2. In part 2, the vehicle decelerates from its highest speed to a halt; we are given $v_0 = 20$ m/s; $v = 0$ and $a = -1.0$ m/s^2 (negative because the acceleration vector points opposite to the direction of motion). (**a**) From Table 2-1, we find t_1 (the duration of part 1) from $v = v_0 + at$. In this way, $20 = 0 + 2.0t_1$ yields $t_1 = 10$ s. We obtain the duration t_2 of part 2 from the same equation. Thus, $0 = 20+(-1.0)t_2$ leads to $t_2 = 20$ s, and the total is $t = t_1 + t_2 = 30$ s.
(**b**) For part 1, taking $x_0 = 0$, we use the equation $v^2 = v_0^2 + 2a(x - x_0)$ from Table 2-1 and find

$$x = \frac{v^2 - v_0^2}{2a} = \frac{20^2 - 0}{2(2.0)} = 100\ \text{(m)}.$$

This position is then the *initial* position for part 2, so that when the same equation is used in part 2 we obtain

$$x - 100 = \frac{v^2 - v_0^2}{2a} = \frac{(0)^2 - (20)^2}{2(-1.0)} .$$

Thus, the final position is $x = 300$ m. That this is also the total distance traveled should be evident (the vehicle did not "backtrack" or reverse its direction of motion).

26. The acceleration is found from Eq. 2-11 (or, suitably interpreted, Eq. 2-7).

$$a = \Delta v / \Delta t = (1020\ \text{km/h})/(1.4\ \text{s})$$
$$= (1020\ \text{m})/(3.6\times1.4\ \text{s}^2) = 202.4\ \text{m/s}^2.$$

In terms of the gravitational acceleration g, this is expressed as a multiple of 9.8 m/s^2 as follows:

$$a = (202.4/9.8)g = 20.6\,g = 21\,g.$$

30. (**a**) Eq. 2-15 is used for part 1 of the trip and Eq. 2-18 is used for part 2:

$$\Delta x_1 = v_{o1}\,t_1 + (½)a_1\,t_1^2,$$

where $a_1 = 2.25$ m/s^2 and $\Delta x_1 = 900/4$ m,

$$\Delta x_2 = v_2\,t_2 - (½)a_2\,t_2^2,$$

where $a_2 = -0.75$ m/s^2 and $\Delta x_2 = 3(900)/4$ m . In addition, $v_{o1} = v_2 = 0$. Solving these equations for the times and adding the results gives $t = t_1 + t_2 = 56.6$ s. (**b**) Eq. 2-16 is used for part 1 of the trip:

$$v^2 = (v_{o1})^2 + 2a_1\Delta x_1$$
$$= 0 + 2(2.25)(900/4) = 1013\ (\text{m}^2/\text{s}^2),$$

which leads to $v = 31.8$ m/s for the maximum speed.

89AP. We neglect air resistance, which justifies setting $a = -g = -9.8\,\text{m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the stone's motion. We are allowed to use Table 2-1 (with Δx replaced by y) because the ball has constant acceleration motion (and we choose $y_0 = 0$). (**a**) We apply Eq. 2-16 to both measurements, with SI units understood.

$$v_B^2 = v_0^2 - 2gy_B \quad\Rightarrow\quad (v/2)^2 + 2g(y_A+3) = v_0^2,$$
$$v_A^2 = v_0^2 - 2gy_A \quad\Rightarrow\quad v^2 + 2gy_A = v_0^2.$$

We equate the two expressions that each equal v_0^2 and obtain

$$\frac{1}{4}v^2 + 2gy_A + 6g = v^2 + 2gy_A \;\Rightarrow\; 6g = \frac{3}{4}v^2 ,$$

which yields $v = (8g)^{1/2} = 8.85\,\text{m/s}$. (**b**) An object moving upward at A with speed v = 8.85 m/s will reach a maximum height $y - y_A = v^2/2g = 4.00\,\text{m}$ above point A (this is again a consequence of Eq. 2-16, now with the "final" velocity set to zero to indicate the highest point). Thus, the top of its motion is 1.00 m above point B.

93AP. We neglect air resistance, which justifies setting $a = -g = -9.8\,\text{m/s}^2$ (taking *down* as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. When something is thrown straight up and is caught at the level it was thrown from (with a trajectory similar to that shown in Fig. 2-25), the time of flight t is half of its time of ascent t_a, which is given by Eq. 2-18 with $\Delta y = H$ and v = 0 (indicating the maximum point).

$$H = vt_a + \frac{1}{2}gt_a^2 \;\Rightarrow\; t_a = \sqrt{2H/g}\,.$$

Writing these in terms of the total time in the air $t = 2t_a$ we have

$$H = \frac{1}{8}gt^2 \Rightarrow\; t = 2\sqrt{2H/g}\,.$$

We consider two throws, one to height H_1 for total time t_1 and another to height H_2 for total time t_2, and we set up a ratio:

$$\frac{H_2}{H_1} = \frac{gt_2^2/8}{gt_1^2/8} = \left(\frac{t_2}{t_1}\right)^2$$

from which we conclude that if $t_2 = 2t_1$ (as is required by the problem) then $H_2 = 2^2H_1 = 4H_1$.

116AP. There is no air resistance, which makes it quite accurate to set $a = -g = -9.8\,\text{m/s}^2$ (where downward is the $-y$ direction) for the duration of the fall. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion; in fact, when the acceleration changes (during the process of catching the ball) we will again assume constant acceleration conditions; in this case, we have $a_2 = +25g = 245\,\text{m/s}^2$. (**a**) The time of fall is given by Eq. 2-15 with v_0 = 0 and y = 0. Thus,

$$t = \sqrt{2y_0/g} = \sqrt{2(145)/9.8} = 5.44\ (\text{s}).$$

(**b**) The final velocity for its free-fall (which becomes the initial velocity during the catching process) is found from Eq. 2-16 (other eqs. can be used but they would use the result from part (a)).

$$v = -\sqrt{v_0^2 - 2g(y - y_0)} = -\sqrt{2gy_0} = -53.3\ \text{m/s}$$

where the negative root is chosen since this is a downward velocity. Thus, the speed is $|v|$ = 53.3 m/s. (**c**) For the catching process, the answer to part (b) plays the role of an *initial* velocity (v_0 = –53.3 m/s) and the final velocity must become zero. Using Eq. 2-16, we find

$$\Delta y_2 = \frac{v^2 - v_0^2}{2a_2} = \frac{0 - (-53.3)^2}{2(245)} = -5.80\ (\text{m}),$$

where the negative value of Δy_2 signifies that the distance traveled while arresting its motion is downward.

117AP. We neglect air resistance, which justifies setting $a = -g = -9.8\ \text{m/s}^2$ (taking down as the $-y$ direction) for the duration of the motion. We are allowed to use Table 2-1 (with Δy replacing Δx) because this is constant acceleration motion. The ground level is taken to correspond to y = 0. (**a**) With $y_0 = h$ and v_0 replaced with $-v_0$, Eq. 2-16 leads to

$$v = \sqrt{(-v_0)^2 \;-\; 2g\,(y - y_0)} \;=\; \sqrt{v_0^2 \;+\; 2gh}\;.$$

The positive root is taken because the problem asks for the speed (the *magnitude* of the velocity). (**b**) We use the quadratic formula to solve Eq. 2-15 for t, with v_0 replaced with $-v_0$,

$$\Delta y \;=\; -v_0t - \frac{1}{2}gt^2 \;\;\Rightarrow\;\; t = \frac{-v_0 + \sqrt{(-v_0)^2 - 2g\Delta y}}{g}\,,$$

where the positive root is chosen to yield $t > 0$. With y = 0 and $y_0 = h$, this becomes

$$t = \frac{\sqrt{v_0^2 \;+\; 2gh} \;-\; v_0}{g}.$$

(**c**) If it were thrown upward with that speed from height h then (in the absence of air friction) it would return to height h with that same downward speed and would therefore yield the same final speed (before hitting the ground) as in part (a). An important perspective related to this is treated later in the book (in the context of energy conservation). (**d**) Having to travel up before it starts its descent certainly requires more time than in part (b). The calculation is quite similar, however, except for now having $+v_0$ in the equation where we had put in $-v_0$ in part (b). The details follow:

$$\Delta y = v_0t - \frac{1}{2}gt^2 \;\;\Rightarrow\;\; t = \frac{v_0 \;+\; \sqrt{v_0^2 \;-\; 2g\Delta y}}{g}.$$

with the positive root again chosen to yield $t > 0$. With y = 0 and $y_0 = h$, we obtain

$$t = \frac{\sqrt{v_0^2 \;+\; 2gh} \;+\; v_0}{g}.$$

Ex. Pb.**2-97**

(如發現錯誤煩請告知 jyang@mail.ntou.edu.tw, Thanks.)

J1.“交通與物理”，黃定維及黃偉能，物理雙月刊，24卷2期(2004年4月) 334-337。

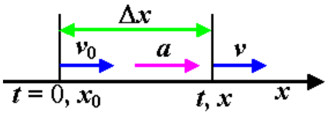
J2. “**The evolution of transport**”, J. H. Ausubel and C. Marchetti, *The Industrial Physicist*, April/ May 2001, pp. 20-24。

第 2 章 直線運動

•一維(x)運動：$x = x(t)$

時間 $t_1 \to 0 \Rightarrow t_2 \to t$

時距$\Delta t \equiv t_2 - t_1 \to t$,



初位置 $x_1 \equiv x(t_1) \to x_0 \Rightarrow$末位置 $x_2 \equiv x(t_2) \to x$,

位移為位置之改變量$\Delta x \equiv x_2 - x_1 \to x - x_0$,

平均速度為單位時間之位移 $v_{av} \equiv \Delta x/\Delta t$,

(瞬時)**速度**為位置之時變率 $v = dx/dt$,

速率指(1)速度大小或(2)總(運動)路程除以時距

初速度 $v_1 \equiv v(t_1) \to v_0 \Rightarrow$末速度 $v_2 \equiv v(t_2) \to v$;

速度改變量$\Delta v \equiv v_2 - v_1 \to v - v_0$,

平均加速度為單位時間之速度改變量 $a_{av} \equiv \Delta v/\Delta t$,

(瞬時)**加速度**為速度之時變率 $a = dv/dt = d^2x/dt^2$,

•一維等加速度運動 $a = a_{av} =$ const.,

$v_{av} = (½)(v_0+v)$, $\Delta v = a\Delta t = at$ or $v = v_0 + at$,

$\Delta x = v_{av}\Delta t = v_0 t + (½)at^2$, or $x = x_0 + v_0 t + (½)at^2$,

$v^2 - v_0^2 = 2a\Delta x$, or $v^2 = v_0^2 + 2a(x-x_0)$,

注意 $v = 0$ 表示運動停止或運動方向即將改變(即前後速度變號), 該點為折返點;

•自由下落運動：鉛直方向之加速度為定值

$x \to y$ and $a \to a_y = -g$ ($\Delta y > 0$: up)

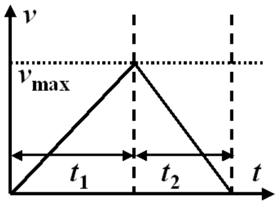
[note $d(x^n)/dx = nx^{n-1}$]

motion,運動; kinematics,運動學; particle,質點; position,位置; origin/zero point,原點; axis,(座標)軸; coordinate,座標; vector,向量; displacement,位移; distance,距離; total distance,總距離; time interval,時距; average velocity,平均速度; (instantaneous) velocity,(瞬時)速度; speed,速率; average acceleration,平均加速度; (instantaneous) acceleration ,(瞬時)加速度; constant acceleration,等加速度, slope,斜率; derivative導數/微商; artery,動脈; tectonic-plate,板塊; fanatical,狂熱的/入迷的; armadillo,犰狳(中南美產); beat-up用壞了的; Porche,保時捷; pickup,臨時湊成的/偶然認識的; NASCAR: National Association of Stock Car Auto Racing,全國運動汽車競賽協會

•上海磁浮電車最高時速— 430 km/h •人類跑百米最快需 9.77 秒(2005/06/14 包威爾) •基隆之 $g = 9.78974$ m/s^2。

•高速鐵路是指每小時行車速度達二百公里以上的行車系統(電車)，台灣高鐵之新幹線電車—平均行車速率 230 km/h = 63.9 m/s，最高時速 300 km/h (台北至高雄左營距 345 km 需 90 分，預定 94.10 底通車，延後一年通車)。

例.設直線賽車比賽總長為 S，若跑車前段加速($a_1 > 0$)，而後段減速($a_2 < 0$)，試計算所花時間。



Sol. Let the distance during the period t_1 of the acceleration a_1 be S_1 and the distance during the period t_2 of the acceleration a_2 be S_2. The maximum speed of car is $v_{max}^2 = 2a_1S_1 = 2|a_2|S_2$. From $S = S_1+S_2$, we have $S_1 = |a_2|S/(a_1+|a_2|) = (½)a_1t_1^2$ and $S_2 = a_1S/(a_1+|a_2|) = (½)|a_2|t_2^2$. Solve them to find $t_1 = \{ 2|a_2|S/[a_1(a_1+|a_2|)] \}^{1/2}$ and $t_2 = \{ 2a_1S/[|a_2|(a_1+|a_2|)] \}^{1/2}$. Using $S = 0.25$ mi = 1320 ft, $a_1 = 24$ ft/s^2, and $a_2 = -32$ ft/s^2 leads to

$$\Delta t = t_1+t_2 = \{2(a_1+|a_2|)S/(a_1|a_2|)\}^{1/2} = 13.9 \text{ s}.$$

馬鞭式創傷：是車禍常見的症狀，主要因為車禍發生時，頭部及頸部的脊骨就如鞭子鞭動一樣，因突然而來的劇動而受傷。

(http://www.chiropractor.com.hk/whiplash_injuries_chi.htm)

Whiplash Injury (鞭樣損傷/頸骨受傷)：鞭樣損傷是指支撐脊柱頸段的軟組織(肌肉及肌腱)受傷，導致頸骨受傷甚至碎裂，令骨與骨之間的盤骨移位，或令脊柱中的神經受損。

病因：頸部受到猛烈震動而向前或向後傾，通常是由於機動車尾端受到碰撞而發生。

病徵：1.頭痛。2.頸部疼痛及僵值。3.手臂及肩膀變得虛弱，甚至癱瘓。4.手部發麻及出現針刺感。5.或會感到焦慮及情緒低落。6.以上症狀或會於意外發生後馬上出現，或於意外後一段時間出現。 (http://www.kingdoctor.com/cgi-bin/content.asp?id=azw5)

治療方法：1.多些休息。2.服食止痛藥止痛。3.若情況仍未見好轉，應盡快求診，確定病情。4.按醫生指示配戴頸箍，以防止頸部移動及承托頸部。5.服食抗炎藥物防止發炎。6.如有需要，應在康復後接受物理治療。